THE

STRENGTH OF MATERIALS.

A TEXT-BOOK

FOR

MANUAL TRAINING SCHOOLS.

BY

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PREFACE.

IN the following pages the attempt is made to give a presentation of the subject of the strength of materials, beams, columns, and shafts, which may be understood by those not acquainted with the calculus. The degree of mathematical preparation required is merely that now given in high schools, and includes only arithmetic, algebra, geometry, and such a course in mechanics as is found in elementary works on physics. In particular the author has had in mind the students in the higher classes of manual training schools, and it has been his aim to present the subject in such an elementary manner that it may be readily comprehended by them and at the same time cover all the essential principles and methods.

As the title implies the book deals mainly with questions of strength, the subject of elastic deformations occupying a subordinate place. As the deductions of the deflections of beams are best made by the calculus they are not here attempted, but the results are stated so that the student may learn their uses; later, if he continues the study of engineering, his appreciation of the proofs that he will then read will be accompanied with true scientific interest.

All the rules for the investigation and design of common beams, including the subject of moment of inertia, are here presented by simple algebraic and geometric

methods. No Greek letters are used, and algebraic operations are made as simple as possible. As the mechanical ideas involved are by far the most difficult part of the subject, a special effort has been made to clearly present them, and to illustrate them by numerous practical numerical examples.

A chapter on the manufacture and general properties of materials is given, as also one on resilience and impact. Problems for students to solve are presented, and it should be strongly insisted upon that these should be thoroughly and completely worked out. It is indeed only by the solution of many numerical exercises that a good knowledge of the theory of the subject can be acquired.

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STRENGTH OF MATERIALS.

CHAPTER I.

ELASTIC AND ULTIMATE STRENGTH.

ART. 1. DIRECT STRESSES.

A 'stress' is an internal resistance which balances an exterior force. If a weight of 500 pounds be suspended by a rope a stress of 500 pounds exists in every cross-section of the rope; or if the rope be cut anywhere and the ends be connected by a spring balance this will register 500 pounds. Stresses are measured in pounds, tons, or kilograms.

A 'unit-stress' is the stress on a unit of area; this is expressed in pounds per square inch or in kilograms per square centimeter. Thus, when a bar of three square inches in cross-section be subject to a pull of 12 000 pounds, the unit-stress is 4 000 pounds per square inch, if the total stress be uniformly distributed over the cross-section.

Three kinds of direct stress are produced by exterior forces which act on a body and tend to change its shape; these are,

Tension, tending to pull apart, as in a rope.

Compression, tending to push together, as in a wall or column.

Shear, tending to cut across, as in punching a plate.

The forces which produce these kinds of stress may be called tensile, compressive, and shearing forces, while the stresses themselves are frequently called tensile, compressive, and shearing stresses.

A ~~stress is~~ always accompanied by a 'deformation' or change of shape of the body. ~~As the applied force~~ increases the deformation ~~and the stress likewise in-crease~~, and if the force be large enough it ~~finally over-comes~~ the stress and the rupture of the body follows.

Tension and compression differ only in regard to direction. A tensile stress in a bar occurs when two forces of equal intensity act upon its ends, each acting

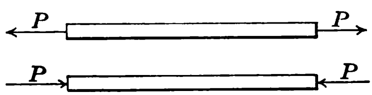

FIG. 1.

away from the end of the bar. In compression the direction of the forces is reversed, each acting toward the end of the bar. The tensile force produces a deformation called 'elongation' and the compressive force produces a deformation called 'shortening.' If P be the force in pounds then the total stress in every section of the bar is equal to P.

Shear implies the action of two forces in parallel planes and very near together, like the forces in a pair of shears, from which analogy the name is derived. Thus, if a bar be laid upon two supports and two loads, each P pounds, be applied to it near the supports, there are hence produced near each support two parallel

forces which tend to cut the bar across vertically. In each of these sections the shearing stress is equal

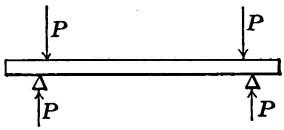

FIG. 2.

to P. The deformation caused by the shearing force P is a vertical sliding between the upward and downward forces; and the bar will be cut across if the external shear overcomes the internal stress.

In all cases of direct stress the total stress P is supposed to be uniformly distributed over the area of the cross-section; this area will be called the 'section area.' Thus if A be the section area and S be the unit-stress, then

$$P = AS, \qquad S = \frac{P}{A}, \qquad A = \frac{P}{S}, \tag{1}$$

from which one of the quantities may be computed when the other two are given. For example, it is known that a wrought-iron bar will rupture under tension when the unit-stress S becomes 50 000 pounds per square inch; if the area A is $4\frac{1}{2}$ square inches, then the tensile force required to rupture the bar is $P = 4\frac{1}{2} \times 50\,000 = 225\,000$ pounds.

Problem 1. A cast-iron bar which is to be subjected to a tension of 34 000 pounds is to be designed so that the unit-stress shall be 2 500 pounds per square inch. What should be the section area in square inches? If the bar is round what should be its diameter? (Ans. $4\frac{11}{32}$ inches.)

ART. 2. THE ELASTIC LIMIT.

When a tensile force is gradually applied to a bar it elongates, and up to a certain limit the elongation is proportional to the force. Thus, if a bar of wrought-iron one square inch in section area and 100 inches long be subjected to a tension of 5 000 pounds it will be found to elongate 0.02 inches; if 10 000 pounds be applied the elongation will be 0.04 inches; if 15 000 pounds it will be 0.06 inches, for 20 000 pounds 0.08 inches, for 25 000 pounds 0.10 inches. Thus far each addition of 5 000 pounds has produced an elongation of 0.02 inches. But when the next 5 000 pounds is added, making a total stress of 30 000 pounds, it will be found that the total elongation is about 0.13 or 0.14 inches, and hence the elongations are increasing more rapidly than the stresses.

The 'elastic limit' is defined to be that unit-stress at which the deformations begin to increase in a faster ratio than the stresses. In the above illustration this limit is about 25 000 pounds per square inch, and this indeed is the average value of the elastic limit for wrought iron. The term 'elastic strength' is perhaps a better word than elastic limit, but the latter is the one in general use.

When the unit-stress in a bar is less than the elastic limit the bar returns, when the stress is removed, to its original length. When the unit-stress is greater than the elastic limit the bar does not fully spring back, but there remains a so-called permanent set. In other words, the elastic properties of a bar are injured if it is stressed beyond the elastic limit. Hence it is a

fundamental rule in designing engineering constructions that the unit-stresses should never exceed the elastic limit of the material.

The following are average values of the elastic limits of the four materials most used in engineering construction under tensile and compressive stresses.

TABLE I. ELASTIC LIMITS.

Material.	Pounds per Square Inch.		Kilos per Sq. Centimeter.	
	Tension.	Compression.	Tension.	Compression.
Timber	3 000	3 000	210	210
Cast Iron	6 000	20 000	420	1 400
Wrought Iron	25 000	25 000	1 750	1 750
Steel	50 000	50 000	3 500	3 500

Those in English units must be carefully kept in mind by the student; and it may be noted that pounds per square inch multiplied by 0.07 will give kilos per square centimeter. But little is known concerning the elastic limit in shear; it is probably about three-fourths of the values above given for tension.

Prob. 2. A square stick of timber is to carry a compressive load of 81 000 pounds. What should be its size in order that the unit-stress may be one-third of the elastic limit? (Ans. 9×9 inches.)

ART. 3. ULTIMATE STRENGTH.

When a bar is under stress exceeding its elastic limit it is usually in an unsafe condition. As the stress is increased by the application of exterior forces the deformation rapidly increases, until finally the rupture of the bar occurs. By the term 'ultimate

strength' is meant that unit-stress which occurs just before rupture, it being the highest unit-stress that the bar will bear.

The ultimate strengths of materials are from two to four times their elastic limits, but for some materials they are much greater in compression than in tension. The average values will be given in subsequent articles.

The 'factor of safety' is a number which results by dividing the ultimate strength by the actual unit-stress that exists in a bar. For example, a stick of timber, 6×6 inches in section area, whose ultimate strength in tension is 10 000 pounds per square inch, is under a tensile stress of 32 400 pounds. The unit-stress then is $32\,400/36 = 900$ pounds per square inch, and the factor of safety is $10\,000/900 = 11$. The factor of safety was formerly much used in designing, but it is now considered the better plan to judge of the security of a body under stress by reference to its elastic limit. Thus in the above case, as the unit-stress is only one-third the elastic limit for timber, the degree of security may be regarded as sufficient.

Prob. 3. A bar of wrought iron $2\frac{1}{2}$ inches in diameter ruptures under a tension of 271 000 pounds. What is its ultimate strength?

(Ans. 55 400 pounds per square inch.)

ART. 4. TENSION.

When a bar is tested under tension, it is done by loads which are gradually applied. The elongations increase proportionally to the stresses until the elastic limit is reached. After the unit-stress has exceeded

the elastic limit the elongations increase more rapidly than the stresses, and this is often accompanied by a reduction in area of the cross-section of the bar. Finally the ultimate strength of the material is reached, and the bar tears apart.

A graphical illustration of these phenomena may be made by laying off the unit-stresses as ordinates and the elongations per unit of length as abscissas. At various intervals, as the test progresses, the applied loads are measured and the resulting elongations are

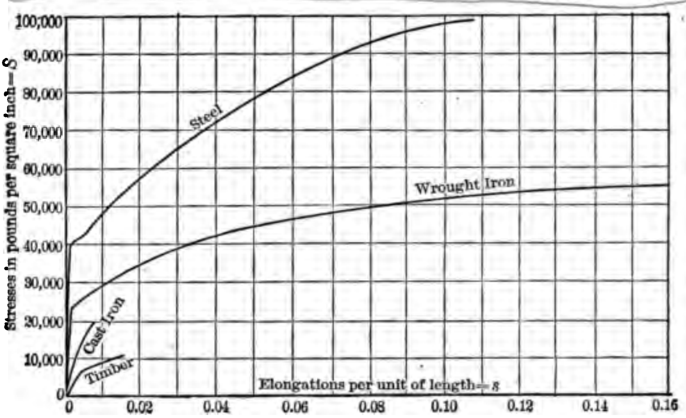

FIG. 3.

measured. The loads divided by the section area give the unit-stresses, while the total elongations divided by the length of the bar give the unit-elongations. On the plot a point is made for the intersection of each unit-stress with its corresponding unit-elongation, and a curve is drawn connecting the several points for each material. In this way curves are plotted showing the properties of each material. It is seen that each curve is a straight line from the origin o until the

elastic limit is reached, showing that the elongations increase proportionally to the unit-stresses. At the elastic limit a sudden change in the curve is seen, and afterwards the elongation increases more rapidly than the stress. The end of the curve indicates the point of rupture. This diagram gives mean comparative values only, and a special curve for any individual test might deviate considerably from the mean form here shown.

The ultimate elongation is an index of the ductility of the material, and is hence generally recorded for wrought iron and steel; this is usually expressed as a percentage of the total length of the bar, or it is 100 times the unit-elongation. The following table gives mean values of the ultimate strengths and ultimate elongations for the principal materials used in tension.

TABLE II. TENSILE STRENGTHS.

Material.	Ultimate Strength.		Ultimate Elongation.
	Pounds per Square Inch.	Kilos per Sq. Centimeter.	Per Cent.
Timber	10 000	700	1.5
Cast Iron	20 000	1 400	0.5
Wrought Iron	55 000	3 850	20
Steel	100 000	7 000	10

All these values should be regarded as rough averages, since they are subject to much variation with different qualities of the material; for instance mild steel beams may be as low as 60 000 while steel wire may be as high as 225 000 pounds per square inch in ultimate tensile strength. The ultimate strengths given in the table should, however, be memorized by the stu-

dent as a basis for future knowledge, and they will be used for all the examples and problems in this book, unless otherwise stated.

Prob. 4. What should be the diameter of a wrought-iron bar so as to carry a tension of 200 000 pounds with a factor of safety of 5? If the bar is cast iron, what should be its diameter? (Ans. $4\frac{13}{16}$ and $7\frac{5}{32}$ inches.)

ART. 5. COMPRESSION.

The phenomena of compression are similar to those of tension provided that the elastic limit is not exceeded, the shortening of the bar being proportional to the applied force. After the elastic limit is passed the shortening increases more rapidly than the stress. If the length of the specimen is less than about ten times its least thickness, failure usually occurs by an oblique splitting or shearing, as seen in the diagram.

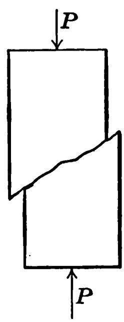

FIG. 4.

If the length be large compared with the thickness, failure usually occurs under a sidewise bending, so that this is not a case of simple compression. All the values given in the following table refer to the

short specimens; longer pieces are called 'columns' or 'struts,' and these will be discussed in Chapter V.

The mean values of the ultimate compressive strengths of the principal materials are tabulated below. These are subject to much variation in different qualities of the materials, but it is necessary for the student to fix them in his mind as a preliminary basis for more extended knowledge. It is seen that

TABLE III. COMPRESSIVE STRENGTHS.

Material.	Ultimate Strength.	
	Pounds per Square Inch.	Kilos per Sq. Centimeter.
Timber	8 000	560
Brick	3 000	210
Stone	6 000	420
Cast Iron	90 000	6 300
Wrought Iron	55 000	3 850
Steel	150 000 .	10 500

timber is not quite as strong in compression as in tension, that cast iron is $4\frac{1}{2}$ times as strong, that wrought iron has the same ultimate strength, and that steel is 50 per cent stronger.

The investigation of a body under compression is made by formula (1) of Art. 1. For example if a stone block 8×12 inches in cross-section is subjected to a compression of 230 000 pounds, the unit-stress produced is $230\,000/96 = 2400$ pounds per square inch, and the factor of safety is $6000/2\,400 = 2\frac{1}{2}$; this is not sufficiently high for stone, as will be seen later.

Prob. 5. A brick $2 \times 4 \times 8$ inches weighs about $4\frac{1}{2}$ pounds. What will be the height of a pile of bricks so

that the unit-stress on the lowest brick shall be one-half of its ultimate strength ? (Ans. 1780 feet.)

ART. 6. SHEAR.

Shearing stresses exist when two forces acting like a pair of shears tend to cut a body between them. When a hole is punched in a plate, the ultimate shearing strength of the material must be overcome. If two thin bars be connected by a rivet and these be subjected to tension, the cross-section of the rivet between the plates is brought into shear. If a bolt is in tension, the forces acting on the head tend to shear or strip it off.

The following table gives the average ultimate shearing strength of different materials as determined

TABLE IV. SHEARING STRENGTHS.

Material.		Ultimate Strength.	
		Pounds per Square Inch.	Kilos per Sq. Centimeter.
Timber	longitudinal	600	42
	transverse	3 000	210
Cast Iron		20 000	1 400
Wrought Iron		50 000	3 500
Steel		70 000	4 900

by experiment. For timber this is much smaller along the grain than across the grain; in the first direction it is called the longitudinal shearing strength, and in the second the transverse shearing strength. Rolled plates of wrought iron and steel, where the process of manufacture induces a fibrous structure, are

also sheared more easily in the longitudinal than in the transverse direction.

Wooden specimens for tensile tests like that shown in the figure will fail by shearing off the ends if the length *ab* is not sufficiently great. For instance, suppose *ab* to be 6 inches, and the diameter of the central part to be 2 inches. The ends are grasped tightly by

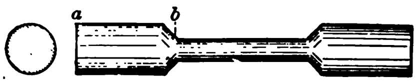

FIG. 5.

the machine and the cross-section of the central part thus brought under tensile stress. The force required to cause rupture by tension is

$$P = AS = 3.14 \times 1^2 \times 10\,000 = 31\,400 \text{ pounds.}$$

But the ends also tend to shear off on the surface of cylinder whose diameter is 2 inches and whose length is *ab*; the force required to cause this rupture by shearing is

$$P = AS = 3.14 \times 2 \times 6 \times 600 = 22\,600 \text{ pounds,}$$

and hence the specimen will fail by shearing off the ends. To prevent this the distance *ab* must be made longer than 6 inches.

Prob. 6. A wrought-iron bolt $1\frac{1}{2}$ inches in diameter has a head $1\frac{1}{4}$ inches long. When a tension of 15 000 pounds is applied to the bolt, find the tensile unit-stress and the factor of safety for tension. Also find the unit-stress tending to shear off the head of the bolt, and the factor of safety against shear. (Ans. Nearly 20.)

ART. 7. WORKING UNIT-STRESSES.

When a body of cross-section A is under a stress P, the unit-stress S produced is found by dividing P by A. By comparing this value of S with the ultimate strengths and elastic limits given in the preceding articles, the degree of security may be inferred. This process is called investigation. The student may not at first be able to form a good judgment with regard to the degree of security, this being a matter which involves some experience as well as acquaintance with engineering precedents and practice. As his knowledge increases, however, his ability to judge whether unit-stresses are or are not too great will constantly improve.

When a body is to be designed to stand a total stress P, the unit-stress S is first assumed in accordance with the rules of practice, and then the section area A is computed. Such assumed unit-stresses are often called working unit-stresses, meaning that these are the unit-stresses under which the material is to act or work. In selecting them, two fundamental rules are to be kept in mind:

1. They should be considerably less than the elastic limits.
2. They should be smaller for sudden stresses than for steady stresses.

The reason for the first requirement is given in Art. 2. The reason for the second requirement is that experience teaches that suddenly applied loads and shocks are more injurious and produce higher unit-stresses than steady loads. Thus a bridge subject

to the traffic of heavy trains must be designed with lower unit-stresses than a roof where the variable load consists only of snow and wind.

It will be best for the student to begin to form his engineering judgment by fixing in mind the following average values of the factors of safety to be used for different materials under different circumstances. The working unit-stress will then be found for any special

TABLE V. FACTORS OF SAFETY.

Material.	For Steady Stress. (Buildings.)	For Varying Stress. (Bridges.)	For Shocks. (Machines.)
Timber	8	10	15
Brick and Stone	15	25	35
Cast Iron	6	15	20
Wrought Iron	4	6	10
Steel	5	7	15

case by dividing the ultimate strengths by these factors of safety. For instance, a short timber strut in a bridge should have a working unit-stress of about $8\,000/10 = 800$ pounds per square inch.

It frequently happens that a designer works under specifications in which the unit-stresses to be used are definitely stated. The writer of the specifications must necessarily be an engineer of much experience and with a thorough knowledge of the best practice. It may be noted also that the particular qualities of timber or steel to be used will influence the selection of working unit-stresses, and in fact different members of a bridge truss are often designed with different unit-

stresses. The two fundamental rules above stated are, however, the guiding ones in all cases.

Modern engineering is the art of economic construction. In numerous instances this will be secured by making all parts of a structure of equal strength, for if one part is stronger than another it has an excess of material which might have been saved.

Prob. 7. A wrought-iron rod is to be under a stress of 82 000 pounds. Find its diameter when it is to be used in a building, and also when it is to be used in a bridge.

(Ans. $2\frac{3}{4}$ and $3\frac{3}{8}$ inches.)

CHAPTER II.

GENERAL PROPERTIES.

ART. 8. AVERAGE WEIGHTS.

The average weights of the six principal materials used in engineering constructions are given in the following table, together with their specific gravities. These are subject to more or less variation, according to the quality of the material. For instance, brick may weigh as low as 100 or as high as 150 pounds per cubic foot, depending upon whether it is soft common quality or fine hard-pressed. Unless otherwise stated, the above values will be used in all the examples and problems in the following pages, and hence those in pounds per cubic foot must be carefully kept in the memory.

TABLE VI. WEIGHT.

Material.	Pounds per Cubic Foot.	Kilos per Cubic Meter.	Specific Gravity.
Timber	40	600	0.6
Brick	125	2 000	2.0
Stone	160	2 600	2.6
Cast Iron	450	7 200	7.2
Wrought Iron	480	7 700	7.7
Steel	490	7 800	7.8

For computing the weights of bars, beams, and

pieces of uniform section area, the following approximate simple rules are much used by engineers:

A wrought-iron bar one square inch in section area and one yard long weighs ten pounds.
Timber is one-twelfth the weight of wrought iron.
Brick is one-fourth the weight of wrought iron.
Stone is one-third the weight of wrought iron.
Cast iron is six per cent lighter than wrought iron.
Steel is two per cent heavier than wrought iron.

For example, if a bar of wrought iron be $1\frac{1}{2} \times 3$ inches in section and 22 feet long, its section area is $4\frac{1}{2}$ square inches and its weight is $45 \times 7\frac{1}{3} = 330$ pounds. A steel bar of the same dimensions will weigh $330 + 0.02 \times 330 = 337$ pounds, and a cast-iron bar will weigh $330 - 0.06 \times 330 = 310$ pounds.

By reversing the above rules the section areas are readily found when the weights per linear yard are given. Thus, if a stick of timber 15 feet long weighs 120 pounds, its weight per yard is 24 pounds and its section area is $2.4 \times 12 = 28.8$ square inches.

Prob. 8. What is the weight of a stone block 12×18 inches and $4\frac{1}{2}$ feet long? How many square inches in the cross-section of a steel railroad rail which weighs 95 pounds per yard? If a cast-iron water pipe 12 feet long weighs 10 000 pounds, what is its section area?

ART. 9. TESTING MACHINES.

The simplest method of testing is by tension, a specimen being used like that shown in Art. 6. The heads are either gripped in jaws, or they are provided with threads so that they may be screwed into nuts to which the forces are applied. The power may be fur-

nished by a lever, a screw, or by hydraulic pressure, the last being the method in machines of high capacity. In these tests the elastic limit, ultimate strength, and the ultimate elongation are generally recorded, the latter being expressed as a percentage of the original length. For ductile materials the contraction of area of the fractured specimen is also noted, as this does not vary with the length of the specimen to the same extent as the ultimate elongation. In such tensile tests the load is applied gradually, and not suddenly or with impact.

The elastic limit is detected by taking a number of measurements of the elongation for different loads, and then noting when these begin to vary more rapidly than the stresses. For ductile materials the change is a sudden one, and it may be often noted by the drop of the scale beam of the machine.

Compressive tests are confined mainly to brick and stone, and are but little used for commercial tests of metals on account of the difficulty of securing a uniform distribution of pressure over the surfaces. Cement, which is always used in compression, is indeed usually tested by tension, this being found to be the cheaper and more satisfactory method.

The capacity of a testing machine is the number of pounds it can exert as tension or compression. A small machine for testing wire or cement need not have a capacity greater than 1 000 or 2 000 pounds. Machines of 50 000, 100 000, and 150 000 pounds for testing metals are common. The Watertown machine has a capacity of 1 000 000 pounds, and can test a small hair or a steel bar of 10 square inches section area with

equal precision. The heaviest machine is that at Phœnixville, Pa., for testing eye-bars, which has a capacity of 2 160 000 pounds.

Tests are also made by loading beams transversely and measuring the deflections, as well as finding the load required to produce rupture. The loads which produce rupture give comparative measures of the ultimate strengths of the beams.

Prob. 9. A steel eye-bar tested at Phœnixville was $10 \times 2\frac{5}{8}$ inches in size and 47 feet long. The length after rupture was 57.6 feet, and the area of the fractured cross-section was 13.0 square inches. Compute the percentage of ultimate elongation and the percentage of reduction of area.
(Ans. 22.5 and 50.5.)

ART. 10. TIMBER.

Good timber is of uniform color and texture, free from knots, sap wood, wind shakes, and decay. It should be well seasoned, which is best done by exposing it to the sun and wind for two or three years to dry out the sap. The heaviest timber is usually the strongest; also the darker the color and the closer the annular rings the stronger and better it is, other things being equal. The strength of timber is always greatest in the direction of the grain, the sidewise resistance to tension or compression being scarcely one-fourth of the longitudinal.

The following table which gives average values of the ultimate strength of a few of the common kinds of timber will be useful for reference. These values have been determined from tests of small specimens carefully selected and dried. Large pieces of timber such as

TABLE VII. STRENGTH OF TIMBER.

Kind.	Pounds per Cubic Foot.	Pounds per Square Inch.	
		Tensile Strength.	Compressive Strength.
Hemlock	25	8 000	5 000
White Pine	27	8 000	5 500
Chestnut	40	12 000	5 000
Red Oak	42	9 000	6 000
Yellow Pine	45	15 000	9 000
White Oak	48	12 000	8 000

are actually used in engineering structures will probably have an ultimate strength of from fifty to eighty per cent of these values. Moreover the figures are liable to a range of 25 per cent on account of variations in quality and condition arising from place of growth, time when cut, and method of seasoning. To cover these variations the factor of safety of 10 is not too high, even for stready stresses.

The shearing strength of timber is still more variable than the tensile or compressive resistance. White pine across the grain may be put at 2 500 pounds per square inch, and along the grain at 500. Chestnut has 1 500 and 600 respectively, yellow pine and oak perhaps 4 000 and 600 respectively.

The elastic limit of timber is poorly defined. In precise tests on good specimens it is sometimes observed at about one-half the ultimate strength, but under ordinary conditions it is safer to put it at one-third. The ultimate elongation is small, usually being between 1 and 2 per cent.

Prob. 10. If a piece of white cedar 2×2 inches in

cross-section ruptures under a compression of 20 800 pounds, what is the size of a square section that will stand 25 000 pounds with a factor of safety of 10?

ART. 11. BRICK.

Brick is made of clay which consists mainly of silicate of alumina with compounds of lime, magnesia, and iron. The clay is prepared by cleaning it carefully from pebbles and sand, mixing it with about one-half its volume of water, and tempering it by hand stirring or in a pug mill. It is then moulded in rectangular boxes by hand or by special machines, and the green bricks are placed under open sheds to dry. These are piled in a kiln and heated for nearly two weeks until those nearest to the fuel assume a partially vitrified appearance.

Three qualities of brick are taken from the kiln; 'arch-brick' are those from around the arches where the fuel is burned, these are hard and often brittle; 'body-brick,' from the interior of the kiln, are of the best quality; 'soft brick,' from the exterior of the pile, are weak and only suitable for filling. Paving brick are burned in special kilns, often by natural gas or by oil, the rate of heating being such as to insure toughness and hardness.

The common size is $2 \times 4 \times 8\frac{1}{4}$ inches, and the average weight $4\frac{1}{2}$ pounds. A pressed brick, however, may weigh nearly $5\frac{1}{2}$ pounds. Good bricks should be of regular shape, have parallel and plane faces, with sharp angles and edges. They should be of uniform texture, and when struck a quick blow should give a sharp metallic ring. The heavier the brick, other things being equal, the stronger and better it is.

Poor brick will absorb when dry from 20 to 30 per cent of its weight of water, ordinary qualities absorb from 10 to 20 per cent, while hard paving brick should not absorb more than 2 or 3 per cent. An absorption test is valuable in measuring the capacity of brick to resist the disintegrating action of frost, and as a rough general rule the greater the amount of water absorbed the less is the strength and durability.

The crushing strength of brick is variable; while a mean value may be 3 000 pounds per square inch, soft brick will scarcely stand 500, pressed brick may run to 10 000, and the best qualities of paving brick have given 15 000 pounds per square inch, or even more. Crushing tests are difficult and expensive to make on account of the labor of preparing the specimen cubes so as to secure surfaces truly parallel. Tensile and shearing tests of bricks are rarely made and but little is known of their behavior under such stresses; the ultimate tensile strength may perhaps range from 50 to 500 pounds per square inch.

Prob. 11. Compute the unit-stress at the base of a brick wall 17 inches thick and 55 feet high. What is the factor of safety?

ART. 12. STONE.

Sandstone, as its name implies, is sand, usually quartzite, which has been consolidated under heat and pressure. It varies much in color, strength, and durability, but many varieties form most valuable building material. In general it is easy to cut and dress, but the variety known as Potsdam sandstone is very hard in some localities.

Limestone is formed by consolidated marine shells, and is of diverse quality. Marble is limestone which has been reworked by the forces of nature so as to expel the impurities, and leave a nearly pure carbonate of lime; it takes a high polish, is easily cut, and makes one of the most beautiful building stones.

Granite is a rock of aqueous origin metamorphosed under heat and pressure; its composition is quartz, feldspar, and mica, but in the variety called gneiss the mica is replaced by hornblende. It is fairly easy to work, usually strong and durable, and some varieties will take a high polish.

Trap, or basalt, is an igneous rock without cleavage. It is hard and tough, and less suitable for building constructions than other rocks, as large blocks cannot be readily obtained and cut to size. It has, however, a high strength, and is remarkable for durability.

The average weight of sandstone is about 150, of limestone 160, of granite 165, and of trap 175 pounds per cubic foot. The ultimate compressive strength of sandstone is about 5 000, of limestone 7 000, of granite 12 000, and of trap 16 000 pounds per square inch; these figures refer to small blocks, but the ultimate strength of large blocks is materially smaller.

The quality of a building stone cannot be safely inferred from tests of strength, as its durability depends largely upon its capacity to resist the action of the weather. Hence corrosion and freezing tests, impact tests, and observations of the behavior of stone under conditions of actual use are more important than the determination of crushing strength in a compression machine.

Prob. 12. A stone pier 12 × 30 feet at the base, 8 × 24 feet at the top, and 16½ feet high is to be built at \$6.37 per cubic yard. What is the total cost? (Ans. \$1 057.54.)

ART. 13. CAST IRON.

Cast iron is a modern product, having been first made in England about the beginning of the fifteenth century. Ores of iron are melted in a blast furnace, producing pig iron. The pig iron is remelted in a cupola furnace and poured into moulds, thus forming castings. Beams, columns, pipes, braces, and blocks of every shape required in engineering structures are thus produced.

Pig iron is divided into two classes, foundry pig and forge pig, the former being used for castings and the latter for making wrought iron. Foundry pig has a dark-gray fracture, with large crystals and a metallic luster; forge pig has a light-gray or silver-white fracture, with small crystals. Foundry pig has a specific gravity of from 7.1 to 7.2, and it contains from 6 to 4 per cent of carbon; forge pig has a specific gravity of from 7.2 to 7.4, and it contains from 4 to 2 per cent of carbon. The higher the percentage of carbon the less is the specific gravity, and the easier it is to melt the pig. Besides the carbon there are present from 1 to 5 per cent of other impurities, such as silicon, manganese, and phosphorus.

The properties and strength of castings depend upon the quality of the ores and the method of their manufacture in both the blast and the cupola furnace. Cold blast pig produces stronger iron than the hot blast, but it is more expensive. Long continued

fusion improves the quality of the product, as also do repeated meltings. The darkest grades of foundry pig make the smoothest castings, but they are apt to be brittle; the light-gray grades make tough castings, but they are apt to contain blow holes or imperfections.

The percentage of carbon in cast iron is a controlling factor which governs its strength, particularly that percentage which is chemically combined with the iron. As average values for the ultimate strength of cast iron, 20 000 and 90 000 pounds per square inch in tension and compression respectively are good figures. In any particular case, however, a variation of from 10 to 20 per cent from these values may be expected, owing to the great variation in quality. The elastic limit is poorly defined, there being no sudden increase in deformation, as in ductile materials.

The high compressive strength and cheapness of cast iron render it a valuable material for many purposes; but its brittleness, low tensile strength and ductility forbid its use in structures subject to variations of load or to shocks. Its ultimate elongation being scarcely one per cent, the work required to cause rupture in tension is small compared to that for wrought iron and steel, and hence as a structural material the use of cast iron must be confined entirely to cases of compression.

Prob. 13. A cast-iron bar weighing 31 pounds per linear yard is to be subjected to tension. How many pounds are required to rupture it? (Ans. 65 700.)

ART. 14. WROUGHT IRON.

The ancient peoples of Europe and Asia were acquainted with wrought iron and steel to a limited extent. It is mentioned in Genesis, iv. 22, and in one of the oldest pyramids of Egypt a piece of iron has been found. It was produced, probably, by the action of a hot fire on very pure ore. The ancient Britons built bloomaries on the tops of high hills, a tunnel opening toward the north furnishing a draft for the fire, which caused the carbon and other impurities to be expelled from the ore, leaving behind nearly pure metallic iron.

Modern methods of manufacturing wrought iron are mainly by the use of forge pig (Art. 13), the one most extensively used being the puddling process. Here the forge pig is subjected to the oxidizing flame of a blast in a reverberatory furnace, where it is formed into pasty balls by the puddler. A ball taken from the furnace is run through a squeezer to expel the cinder and then rolled into a muck bar. The muck bars are cut, laid in piles, heated, and rolled, forming what is called merchant bar. If this is cut, piled, and rolled again, a better product, called best iron, is produced. A third rolling gives 'best best' iron, a superior quality, but high in price.

The product of the rolling-mill is bar iron, plate iron, shape iron, beams, and rails. Bar iron is round, square, and rectangular in section; plate iron is from $\frac{1}{4}$ to 1 inch thick, and of varying widths and lengths; shape iron includes angles, tees, channels, and other forms used in structural work; beams are I-shaped,

and of the deck or rail form. Structural shapes and beams are, however, now more extensively rolled in mild steel than in wrought iron.

Wrought iron is metallic iron containing less than 0.25 per cent of carbon, and which has been manufactured without fusion. Its tensile and compressive strengths are closely equal, and range from 50 000 to 60 000 pounds per square inch. The elastic limit is well defined at about 25 000 pounds per square inch, and within that limit the law of proportionality of stress to deformation is strictly observed. It is tough and ductile, having an ultimate elongation of from 20 to 30 per cent. It is malleable, can be forged and welded, and has a high capacity to withstand the action of shocks. It cannot, however, be tempered, nor can it be melted, except by the highest temperatures.

The cold-bend test for wrought iron is an important one for judging of general quality. A bar perhaps $\frac{3}{4} \times \frac{3}{4}$ inches and 15 inches long is bent when cold either by pressure or by blows of a hammer. Bridge iron should bend through an angle of 90 degrees to a curve whose radius is twice the thickness of the bar, without cracking. Rivet iron should bend through 180 degrees until the sides of the bar are in contact, without showing signs of fracture. Wrought iron that breaks under this test is lacking in both strength and ductility.

The process of manufacture, as well as the quality of the pig iron, influences the strength of wrought iron. The higher the percentage of carbon the greater is the strength. Best iron is 10 per cent stronger than or-

dinary merchant iron owing to the influence of the second rolling. Cold rolling causes a marked increase in elastic limit and ultimate strength, but a decrease in ductility or ultimate elongation. Annealing lowers the ultimate strength, but increases the elongation. Iron wire, owing to the process of drawing, has a high tensile strength, sometimes greater than 100 000 pounds per square inch.

Good wrought iron when broken by tension shows a fibrous structure. If, however, it be subject to shocks or to repeated stresses which exceed the elastic limit, the molecular structure becomes changed so that the fracture is more or less crystalline. The effect of a stress slightly exceeding the elastic limit is to cause a small permanent set, but the elastic limit will be found to be higher than before. This is decidedly injurious to the quality of the material on account of the accompanying change in structure, and hence it is a fundamental principle that the working unit-stresses should not exceed the elastic limit. For proper security indeed the allowable unit-stress should seldom be greater than one-half the elastic limit.

In a rough general way the quality of wrought iron may be estimated by the product of its tensile strength and ultimate elongation, this product being an approximate measure of the work required to produce rupture. Thus high tensile strength is not usually a good quality when accompanied by a low elongation.

Prob. 14. What should be the length of a wrought-iron bar, so that when hung at its upper end it will rupture there under the stress produced by its own weight ?

(Ans. 16 500 feet.)

ART. 15. STEEL.

Steel was originally produced directly from pure iron ore by the action of a hot fire, which did not remove the carbon to a sufficient extent to form wrought iron. The modern processes, however, involve the fusion of the ore, and the definition of the United States law is that "steel is iron produced by fusion by any process, and which is malleable." Chemically, steel is a compound of iron and carbon generally intermediate in composition between cast and wrought iron, but having a higher specific gravity than either. The following comparison points out the distinctive differences between the three kinds of iron:

	Per cent of Carbon.	Spec. Grav.	Properties.
Cast Iron,	5 to 2	7.2	Fusible, not malleable.
Steel,	1.50 to 0.10	7.8	Fusible and malleable.
Wrought Iron,	0.30 to 0.05	7.7	Malleable, not fusible.

It should be observed that the percentage of carbon alone is not sufficient to distinguish steel from wrought iron; also, that the mean values of specific gravity stated are in each case subject to considerable variation.

The three principal methods of manufacture are the crucible process, the open-hearth process, and the Bessemer process. In the crucible process impure wrought iron or blister steel, with carbon and a flux, are fused in a sealed vessel to which air cannot obtain access; the best tool-steels are thus made. In the open-hearth process pig iron is melted in a Siemens furnace, wrought-iron scrap being added until the proper degree of carbonization is secured. In the Bessemer process pig iron is completely decarbonized in a converter by

an air blast and then recarbonized to the proper degree by the addition of spiegeleisen. The metal from the open-hearth furnace or from the Bessemer converter is cast into ingots, which are rolled in mills to the required forms. The open-hearth process produces steel for guns, armor plates, and for some structural purposes; the Bessemer process produces steel for railroad rails and also for structural shapes.

The physical properties of steel depend both upon the method of manufacture and upon the chemical composition, the carbon having the controlling influence upon strength. Manganese promotes malleability and silicon increases the hardness, while phosphorus and sulphur tend to cause brittleness. The higher the percentage of carbon within reasonable limits the greater is the ultimate strength and the less the elongation.

A classification of steel according to the percentage of carbon and its physical properties of tempering and welding is as follows:

Extra hard,	1.00 to 0.60% C., takes high temper, but not weldable.
Hard,	0.70 to 0.40% C., temperable, but welded with difficulty.
Medium,	0.50 to 0.20% C., poor temper, but weldable.
Mild,	0.40 to 0.05% C., not temperable, but easily welded.

It is seen that these classes overlap so that there are no distinct lines of demarcation. The extra-hard steels are used for tools, the hard steels for piston-rods and other parts of machines, the medium steels for rails, ties, and guns, and the mild or soft steels for beams and structural purposes.

The structural steel used in bridges and buildings has an ultimate tensile strength of from 60 000 to

70 000 pounds per square inch, with an elastic limit from 30 000 to 40 000 pounds per square inch. The hard and extra-hard steels are much higher in strength. By the use of nickel as an alloy steel has been made with an ultimate tensile strength of 277 000 and an elastic limit of over 100 000 pounds per square inch.

The compressive strength of steel is always higher than the tensile strength. The maximum value recorded for hardened steel is 392 000 pounds per square inch. The expense of commercial tests of compression is, however, so great that they are seldom made. The shearing strength is about three-fourths of the tensile strength.

Steel castings are extensively used for axle-boxes, cross-heads, and joints in structural work. They contain from 0.25 to 0.50 per cent of carbon, ranging in tensile strength from 60 000 to 100 000 pounds per square inch.

Steel has entirely supplanted wrought iron for railroad rails, and largely so for structural purposes. Its price being the same, its strength greater, its structure more homogeneous, the low and medium varieties are coming more and more into use as a satisfactory and reliable material for large classes of engineering constructions.

Prob. 15. A short steel piston-rod is to be designed to be used with a piston which is 20 inches in diameter and subject to a steam-pressure of 150 pounds per square inch. If the ultimate strength of the steel is 90 000 pounds per square inch, what should be its diameter, allowing the proper factor of safety? (Ans. $3\frac{1}{8}$ inches.)

ART. 16. OTHER MATERIALS.

Common mortar is composed of one part of lime to five parts of sand by measure. When six months old its tensile strength is from 15 to 30, and its compressive strength from 150 to 300 pounds per square inch. Its strength slowly increases with age, and it may be considerably increased by using a smaller proportion of sand.

Hydraulic mortar is composed of hydraulic cement and sand in varying proportions. The less the proportion of sand the greater is its strength. A common proportion is 3 parts sand to 1 of cement, the strength of this being about one-fourth of the neat cement. The Rosendale cements are of lighter color, lower weight, and lesser strength than the Portland cement, but they are quicker in setting and cheaper in price. When one week old, neat Rosendale cement has a tensile strength of about 125 and Portland cement about 300 pounds per square inch; when one year old the tensile strengths are about 300 and 500 pounds per square inch respectively. The compressive strength is from 8 to 10 times the tensile strength, and it increases more rapidly with age.

Concrete, composed of hydraulic mortar and broken stone, is an ancient material, having been extensively used by the Romans. It is mainly employed for foundations and monolithic structures, but in some cases large blocks have been made which are laid together like masonry. Like mortar, its strength increases with age. When six months old its mean compressive strength ranges from 700 to 1500 pounds per square

inch, and when one year old it is probably about fifty per cent greater.

Ropes are made of hemp, of manilla, and of iron or steel wire with a hemp center. A hemp rope one inch in diameter has an ultimate strength of about 6 000 pounds, and its safe working strength is about 800 pounds. A manilla rope is slightly stronger. Iron and steel ropes one inch in diameter have ultimate strengths of about 36 000 and 50 000 pounds respectively, the safe working strengths being 6 000 and 8 000 pounds. As a fair rough rule, the strength of ropes may be said to increase as the squares of their diameters.

Aluminum is a silver-gray metal which is malleable and ductile and not liable to corrode. Its specific gravity is about 2.65, so that it weighs only 168 pounds per cubic foot. Its ultimate tensile strength is about 25 000 pounds per square inch, and its ultimate elongation is also low. Alloys of aluminum and copper have been made with a tensile strength and elongation exceeding those of wrought iron, but have not come into use as structural materials.

Prob. 16. Ascertain the weight of lead and brass per cubic foot, and their ultimate tensile strengths.

CHAPTER III.

MOMENTS FOR BEAMS.

ART. 17. THE PRINCIPLE OF MOMENTS.

The moment of a force with respect to a point is a quantity which measures the tendency of the force to cause rotation about that point. The moment is the product of the force by the length of its lever-arm, the lever-arm being a line drawn from the point perpendicular to the direction of the force. Thus if P be any

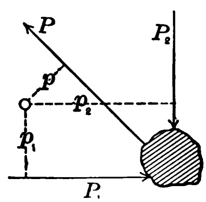

FIG. 6.

force and p the length of a perpendicular drawn to it from any point, the product Pp is the moment of the force with respect to that point. As P is in pounds and p is in feet or inches, the moment is a compound quantity which is called pound-feet or pound-inches.

The most important principle in mechanics is the principle of moments. This asserts that if any number of forces in the same plane be in equilibrium, the algebraic sum of their moments about any point in that plane is equal to zero. This principle results from the

meaning of the word equilibrium, which implies that the body on which the forces act is at rest; and since it is at rest the forces taken collectively have no tendency to turn it around any point. All experience teaches that the principle of moments is, indeed, a law of nature whose truth is universal.

The point from which the lever-arms are measured is often called the 'center of moments.' Forces which tend to turn around this center in the direction of motion of the hands of a watch have positive moments, and those which tend to turn in the opposite direction have negative moments. Thus, in the above figure, the numerical values of Pp and P_1p_1 are negative, while that of P_2p_2 is positive. If the forces be in equilibrium, the sum $Pp + P_1p_1$ has the same numerical value as P_2p_2, or the algebraic sum of the three moments is zero; this will be the case wherever the center of moments be taken.

In all investigations regarding the strength of beams, the principle of moments is of constant application. A beam is a body held in equilibrium by the downward loads and the upward pressures of the supports. As the beam is at rest these forces are in equilibrium, and the algebraic sum of their moments is zero about any point in the plane. Moreover by further use of the principle of moments the stresses in all parts of the beam due to the given loads may be determined.

Prob. 17. In the above figure the three forces are in equilibrium, P_1 being 500 pounds, P_2 being 866 pounds, and the lever-arms being $p = 1.5$ feet, $p_1 = 3.5$ feet, $p_2 = 5.1$ feet. Show that the force P is 1 777 pounds.

ART. 18. REACTIONS OF SUPPORTS.

Let a simple beam resting on two supports near its ends be subject to a load P situated at 6 feet from the left support, and let the span be 24 feet. Taking the

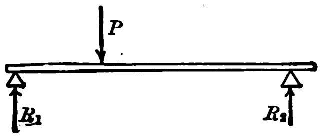

FIG. 7.

center of moments at the right support, the lever-arm of R_1 is 24 feet, that of P is 18 feet, and that of R_2 is 0; then by the application of the principle of moments $R_1 \times 24 - P \times 18 = 0$, or $R_1 = \frac{3}{4}P$. Again, taking the center of moments at the left support the lever-arm of R_1 is 0, that of P is 6 feet, and that of R_2 is 24 feet; then likewise from the principle of moments $-R_2 \times 24 + P \times 6 = 0$, or $R_2 = \frac{1}{4}P$. The sum of these two reactions is equal to P, as should of course be the case.

The reactions caused by the weight of the beam itself may be found in a similar manner, the uniform load being supposed concentrated at its center of gravity in stating the equations of moments. Thus, if the weight of the beam be W, the two equations of moments are found to be $R_1 \times 24 - W \times 12 = 0$, and $-R_2 \times 24 + W \times 12 = 0$, from which $R_1 = \frac{1}{2}W$ and $R_2 = \frac{1}{2}W$.

The reactions due to both uniform and concentrated loads on a simple beam may also be computed in one operation. As an example, let there be a simple beam 12 feet long between the supports and weighing 35

pounds per linear foot, its total weight being 420 pounds. Let there be three loads of 300, 60, and 150 pounds, placed 3, 5, and 8 feet respectively from the left support. To find the left reaction R_1, the center

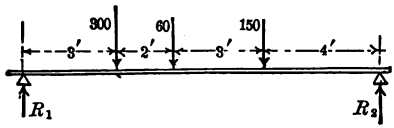

FIG. 8.

of moments is taken at the right support and the weight of the beam regarded as concentrated at its middle: then the equation of moments is

$$R_1 \times 12 - 420 \times 6 - 300 \times 9 - 60 \times 7 - 150 \times 4,$$

from which $R_1 = 520$ pounds. In like manner, to find R_2 the center of moments is taken at the left support, and then

$$-R_2 \times 12 + 420 \times 6 + 300 \times 3 + 60 \times 5 + 150 \times 8,$$

from which $R_2 = 410$ pounds. As a check the sum of R_1 and R_2 is found to be 930 pounds, which equals the weight of the beam and the three loads.

By means of the principle of moments other problems relating to reactions of beams may also be solved. For instance, if a simple beam 12 feet long weighs 30 pounds per linear foot and carries a load of 600 pounds, where should this load be put so that the left reaction may be twice as great as the right reaction? Here let x be the distance from the left support to the load; let R_1 be the left reaction and R_2 the right reaction. Then taking the centers of moments at the right and left support in succession there are found

$$R_1 = 180 + 50(12 - x), \quad R_2 = 180 + 50x,$$

and placing R_1 equal to $2R_2$ there results $x = 2.8$ feet.

Prob. 18. A beam weighing 30 pounds per linear foot rests upon two supports 18 feet apart. A weight of 700 pounds is placed at 5 feet from the left end, and one of 500 pounds is placed at 8 feet from the right end. Find the reactions due to the total load.

ART. 19. BENDING MOMENTS.

The 'bending moment' at any section of a beam is the algebraic sum of the moments of all the vertical forces on the left of that section. It is a measure of the tendency of those forces to cause rotation around that point. At the ends of a simple beam there are no bending moments, but at all other sections they exist, and the greater the bending moment the greater are the horizontal stresses in the beam, these stresses in fact being produced by the bending moment.

For example, let a beam 30 feet long have three loads of 100 pounds each, situated at distances of 8, 12, and 22 feet from the left support. By the method of

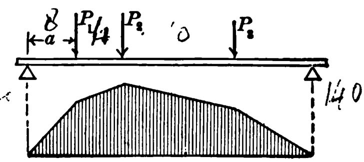

FIG. 9.

the previous article the left reaction R_1 is 160 pounds and the right reaction R_2 is 140 pounds. For a section 4 feet from the left support the bending moment is $160 \times 4 = 640$ pound-feet, and for a section at 8 feet from the left support the bending moment is $160 \times 8 = 1\,280$ pound feet. For a section 10 feet from the left support there are two vertical forces on the left of the

section, 160 acting up and 100 acting down, so that the bending moment is $160 \times 10 - 100 \times 2 = 1\,400$ pound-feet. For a section at the middle of the beam the bending moment is $160 \times 15 - 100 \times 7 - 100 \times 3 = 1\,400$ pound-feet. For a section under the third load the bending moment is, in like manner, 1 120 pound-feet, and for a section at 3 feet from the right support it is 420 pound-feet. The vertical ordinates underneath the beam represent the values of these bending moments, and the diagram thus formed shows how the bending moments vary throughout the length of the beam.

For a simple beam of span l and uniformly loaded with w pounds per linear unit, each reaction is $\frac{1}{2}wl$. For any section distant x from the left support, the

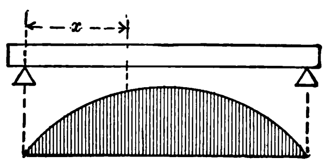

FIG. 10.

bending moment is $\frac{1}{2}wl \times x - wx \times \frac{1}{2}x$, where the lever-arm of the reaction is x and the lever-arm of the load wx is $\frac{1}{2}x$. If w be 80 pounds per linear foot and l be 30 feet, the bending moment at any section is then $1\,200\,x - 40\,x^2$. For $x = 10$ feet, the bending moment is 8 000 pound-feet; for $x = 15$ feet it is 9 000 pound-feet; for $x = 20$ feet it is 8 000 pound-feet, and so on. The diagram shows the distributions of moments throughout the beam, and it can be demonstrated that the curve joining the ends of the ordinates is the common parabola.

When a beam is loaded both uniformly and with concentrated loads, the bending moments for all sections may be found in a similar manner. The maximum bending moment indicates the point where the beam is under the greatest horizontal stresses; this will usually be found near the middle of the beam and often under one of the concentrated loads. For simple beams resting on two supports at their ends all the bending moments are positive. It may further be noted that if the vertical forces on the right of the section be used the same numerical values will be found for the bending moments.

Prob. 19. A simple beam of 12 feet span weighs 60 pounds per linear foot, and has a load of 125 pounds at 4 feet from the left end. Compute the bending moments for sections distant 2, 4, 6, 8, 10 feet from the left support, and construct the diagram of bending moments.

ART. 20. RESISTING MOMENTS.

Suppose a simple beam to be cut by an imaginary vertical plane MN and the portion on the right of that plane to be removed. In order that the remaining

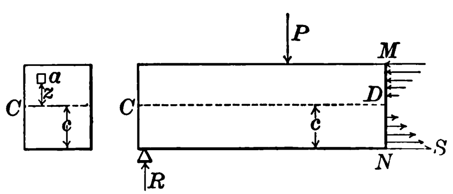

FIG. 11.

part may be in equilibrium, forces must be applied to the section, in the figure horizontal forces are shown, and these represent the horizontal stresses in the sec-

tion. It is found by experiment that there is a certain line CD on the side of the beam which does not change in length under the bending, and hence there is no horizontal stress upon it. Below this neutral line the fibers in a simple beam are found to be elongated and above it they are shortened; thus the stresses below the neutral line are tension and those above it are compression.

The reaction and loads on the left of the section MN together with the stresses acting on that section constitute a system of forces in equilibrium. The algebraic sum of the moments of the reaction and loads with respect to the point D is the bending moment for the section MN, the value of which may be found by the methods of the last article. This bending moment causes a bending which is resisted by the horizontal stresses in the section.

A neutral line like CD is also found in all longitudinal vertical sections of the beam. There is in fact a 'neutral surface' extending throughout the entire width of the beam, and the intersection of this neutral surface with any section area gives a line CC which is called the 'neutral axis' of that section. It is found by experiment that the horizontal stresses increase uniformly from the neutral axis to the top and bottom of the beam, provided that the elastic limit of the material be not exceeded. Thus, if S be the unit-stress at the upper or lower side of the beam, the unit-stress half-way between that side and the neutral axis is $\frac{1}{2}S$. Also if c be the distance from the neutral axis to the upper or lower side of the beam, and z be any distance less than c, then the unit-stress at the distance z is $S\frac{z}{c}$.

'Resisting moment' is the term used to denote the algebraic sum of the moments of all the horizontal stresses in a section with respect to its neutral axis. Let a be any small elementary area of the cross-section at a distance z from the neutral axis. The unit-stress on this small area is $S\frac{z}{c}$, and hence the total stress on this small area is $S\frac{az}{c}$. The moment of this stress with respect to the point D is $S\frac{az}{c}$ multiplied by its lever-arm z, or $\frac{Saz^2}{c}$. The resisting moment is the algebraic sum of all the values of $\frac{Saz^2}{c}$ for all possible values of z. Or, letting Σ denote this process of summation,

$$\text{Resisting moment} = \frac{S}{c}\Sigma az^2.$$

Now the quantity Σaz^2 is what is called the 'moment of inertia' of the cross-section, and it will be shown in Art. 22 how its value is found. The moment of inertia is designated by I, and hence

$$\text{Resisting moment} = \frac{SI}{c}.$$

This is a general expression applicable to all kinds of cross-sections, and in the next chapter it will be constantly used in the investigation and design of beams.

The term 'moment of inertia' has no reference to inertia when it is applied to plane surfaces, as is here the case. It is merely a name for the quantity Σaz^2, and this quantity is found by multiplying each elemen-

tary area by the square of its distance from the given axis and taking the sum of the products. As z^2 is always positive whether z be positive or negative, Σaz^2 or the moment of inertia I is always positive.

Prob. 20. In the above figure let MD and DN be each 6 inches and let the width of the beam CC be 8 inches. If the tensile unit-stress S on the bottom of the beam is 600 pounds per square inch, the compressive unit-stress on the top of the beam is also 600 pounds per square inch. Show that the total tensile stress is 14 400 pounds, and that the total compressive stress is also 14 400 pounds.

ART. 21. CENTERS OF GRAVITY.

The center of gravity of a plane surface is that point upon which a thin sheet of cardboard, having the same shape as the given surface, can be balanced when held in a horizontal position. In the investigation of beams its section area is the given surface, and it is required to know the distances from the top or bottom of the section to the center of gravity. The letter c will be used to denote these distances when they are equal, and the longest of these distances when they are unequal.

For a square, rectangle, or circle, whose depth is d, it is evident that $c = \frac{1}{2}d$. Also for a section of **I** shape, where the upper and lower flanges are equal in size, it is plain that $c = \frac{1}{2}d$.

For a **T** section c is greater than $\frac{1}{2}d$, and its value is to be found by using the principle of moments. If the width of the horizontal flange be 4 inches and its thickness $1\frac{1}{4}$ inches, the area of the flange is 5 square inches; if the height of the vertical web be 6

inches and its thickness 1 inch, the area of the web is 6 square inches. The total area of the cross-section is then 11 square inches. Now if this section be a thin

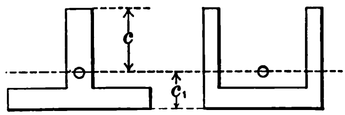

FIG. 12.

sheet held in a horizontal plane the weights of the two parts and the whole are represented by 5, 6, and 11. With respect to an axis at the end of the web the lever-arms of these weights are $6\frac{5}{8}$ inches, 3 inches, and c inches; the equation of moments then is

$$5 \times 6\tfrac{5}{8} + 6 \times 3 - 11 \times c = 0,$$

from which the value of c is found to be 4.65 inches.

The method of moments may thus be applied to areas as well as to forces. If a be any area and z the distance of its center of gravity from an axis, the product az is called the static moment of the area. The sum of the static moments of all parts of the figure is represented by Σaz, and if A be the total section area, then

$$c = \frac{\Sigma az}{A} \qquad (2)$$

is a general expression of the method of finding the distance c. If the axis be taken within the section, some of the z's are negative, and if the axis passes through the center of gravity of the section then the quantity Σaz is zero.

For the channel section ⊓ the same method is to be followed as for the T. When the cross-section is

bounded by curved lines, as in a railroad rail, it is to be divided up into small rectangles and the value of a be found for each; the sum of all the a's is A, and then by the above method the value of c is computed. For the various rolled shapes found in the market the values of c are thus determined by the manufacturers and published for the information of engineers.

Triangular beams are never used, but it is often convenient to remember that for any triangle whose depth is d the value of c is $\frac{2}{3}d$.

Prob. 21. A deck-beam used in buildings has a rectangular flange $4 \times \frac{3}{4}$ inches, a rectangular web $5 \times \frac{1}{2}$ inches, and an elliptical head which is 1 inch in depth and whose area is 1.6 square inches. Find the distance of the center of gravity from the top of the head. (Ans. 4.04 inches.)

ART. 22. MOMENTS OF INERTIA.

The moment of inertia of a plane surface with respect to an axis is the sum of the products obtained by multiplying each elementary area by the square of its distance from that axis. In the discussion of beams the axis is always taken as passing through the center of gravity of the cross-section and parallel to the top and bottom lines of the cross-section. Let I be this moment of inertia, as in Art. 20; its value is to be found by determining the quantity Σaz^2.

To find I for a rectangle of breadth b and depth d, let CC be the axis through the center of gravity and parallel to b. Let the elementary area a be a small strip EE parallel to CC and at a distance z from it. Let a line gh be drawn parallel to the depth d of the

rectangle, and normal to gh let lines be drawn equal to the squares of z; thus ee is the square of CE, and gg is the square of CG. Now the elementary product az^2 is the elementary area EE multiplied by the ordinate ee; hence Σaz^2 is represented by a solid standing on bd whose variable height is shown by the shaded area $ghhcg$. But the volume of this solid is the product of its length b and this shaded area. The curve ceg is a parabola because each line ee is the square of the

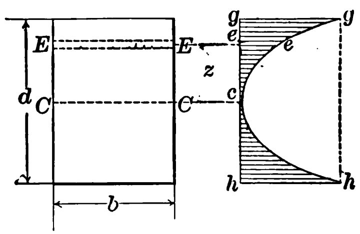

FIG. 13.

corresponding altitude ce; accordingly the shaded area is one-third of $ghhg$. But gh is equal to d, and gg is equal to $(\frac{1}{2}d)^2$; thus the shaded area is represented by $\frac{1}{3} . d . \frac{1}{4}d^2$, or $\frac{1}{12}d^3$. Hence

$$I = b \times \tfrac{1}{12}d^3 = \tfrac{1}{12}bd^3$$

is the moment of inertia of a rectangle about an axis through its center of gravity and parallel to its base.

The moment of inertia is a compound quantity resulting by multiplying an area by the square of a distance; it thus contains the linear unit four times. If $b = 3$ inches and $d = 4$ inches, then $I = 16$ inches^4, or the numerical unit of I is biquadratic inches.

Moments of inertia when referred to the same axis can be added or subtracted like any other qualities

which are of the same kind. Thus, let there be a hollow rectangular section whose outside depth and

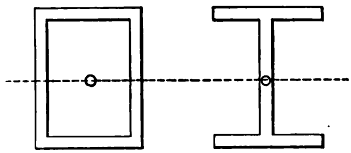

FIG. 14.

breadth are b and d and whose inside depth and breadth are b_1 and d_1, the thickness of the metal being the same throughout. Then the moment of inertia of this section is found by subtracting the moment of inertia of the inner rectangle from that of the outer one, or

$$I = \tfrac{1}{12}bd^3 - \tfrac{1}{12}b_1d_1^3$$

is the moment of inertia for the section whose area is $bd - b_1d_1$.

For the common I beams whose flanges are equal the same method applies. Let b be the width of the flanges and d the total depth; also let t be the thickness of the web and t_1 the thickness of the flanges. The moment of inertia of the area $(b - t)(d - 2t_1)$ is then to be subtracted from the moment of inertia of the area bd, or

$$I = \tfrac{1}{12}bd^3 - \tfrac{1}{12}(b - t)(d - 2t_1)^3$$

is the moment of inertia for the I section.

For the T section the distance c from the end of the web to the axis through the center of gravity must first be computed by the method of the last article. Then c_1, the distance from the outside of the flange to

the axis, is also known. Let b be the breadth of the flange and t_1 its thickness, and let t be the thickness of the web. Then the moment of inertia of the area tc is one-half of that of a rectangle of depth $2c$, or $\frac{1}{2} \times \frac{1}{12}t(2c)^3$,

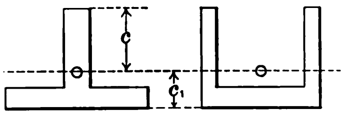

FIG. 15.

which is $\frac{1}{3}tc^3$; also the moment of inertia of the area bc_1 is one-half of that of a rectangle of depth $2c_1$. Adding these together and subtracting the moment of inertia of the area $(b - t)(c_1 - t_1)$, there results

$$I = \tfrac{1}{3}tc^3 + \tfrac{1}{3}bc_1^3 - \tfrac{1}{3}(b - t)(c_1 - t_1)^3,$$

which is the moment of inertia for the T section. The same formula applies to the ⊓ section if t be the thickness of the two webs.

The above formulas for I and T sections are correct for cast-iron beams where the corners are but little rounded. For wrought iron and steel beams, however, the flanges are not usually of uniform thickness, and all the corners are rounded off by curves, so that the formulas are not strictly correct; for such shapes the numerical values of the moments of inertia for all the sections in the market are published by the manufacturers, so that it is not necessary for engineers to compute them. (See Art. 30.)

In this chapter the fundamental applications of moments to be used in discussing beams have been presented, and it is now possible to take up the subject and give the theory of equilibrium of beams clearly

and logically, so that the student may undertake practical problems in the most satisfactory manner.

Prob. 22. A steel I beam weighing 80 pounds per linear foot is 24 inches deep, its flanges being 7 inches wide and $\frac{7}{8}$ inches mean thickness, while the web is 0.5 inches thick. The moment of inertia stated by the manufacturer is 2088 inches^4. Compute it by the formula here given.

(Ans. 2097 inches^4.)

CHAPTER IV.

CANTILEVER BEAMS AND SIMPLE BEAMS.

ART. 23. DEFINITIONS AND PRINCIPLES.

A simple beam is a bar resting upon supports at its ends, and is the kind most commonly in use. A cantilever beam is a bar resting on one support at the middle, or if a part of a beam projects out from a wall or beyond a support this part is called a cantilever beam. In a simple beam the lower part is under tension and the upper part under compression; in a cantilever beam the reverse is the case. Unless otherwise stated, all beams will be regarded as having the section area uniform throughout the entire length.

Since a beam is at rest the internal stresses in any section hold in equilibrium the external forces on each

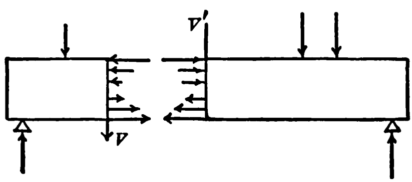

FIG. 16.

side of that section. Thus, if a beam be imagined to be cut apart and the two parts separated, as in the figure, forces must necessarily be required to prevent the parts from falling. These internal forces or stresses may be resolved into horizontal and vertical com-

ponents. The horizontal components are stresses of tension and compression, while the vertical components add together and form a stress known as the resisting shear.

Each side of the beam is held in equilibrium by the vertical forces and stresses that act upon it. The vertical forces are the reaction and the loads, the stresses are the horizontal ones of tension and compression, and the vertical one of shear. The sum of all the horizontal tensile stresses must be equal to the sum of all the horizontal compressive stresses, or otherwise there would be longitudinal motion. The sum V of the vertical stresses must equal the algebraic sum of the reaction and loads, or otherwise there would be motion in an upward or downward direction. Lastly, the sum of the moments of the stresses in the section must equal the sum of the moments of the vertical forces, or otherwise there would be rotation. These statical principles apply to each part into which the beam is supposed to be divided.

It is found by experiment that the fibers on one side of the beam are elongated and on the other shortened, while between is a neutral surface, which is

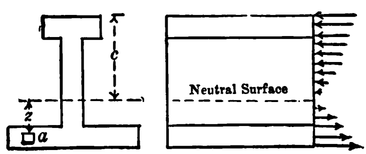

FIG. 17.

unchanged in length. It is also found that the amount of elongation or shortening of any fiber is directly proportional to its distance from the neutral surface. Hence, if the elastic limit be not surpassed, the stresses

are also proportional to their distances from the neutral surface.

From the above it can be shown that the neutral axis passes through the center of gravity of the cross-section. For if S be the unit-stress on the remotest fiber and c its distance from the neutral axis, then the unit-stress at the distance z is $S\frac{z}{c}$, and the total stress on an elementary area a is $S\frac{az}{c}$. The algebraic sum of all the horizontal stresses in the section then is $\frac{S}{c}\Sigma az$. From the above statical principles this sum must be zero, and it hence follows that Σaz must be zero; that is, the sum of the moments of the elementary areas is zero with respect to the neutral axis. Hence the neutral axis passes through the center of gravity, for the center of gravity is that point upon which the surface can be balanced or for which $\Sigma az = 0$.

Prob. 23. An I beam which is 20 feet long weighs 700 pounds and the area of its cross-section is 10.29 square inches. What is the kind of material?

ART. 24. RESISTANCE TO SHEARING.

When a beam is short it sometimes fails by shearing in a vertical section near one of the supports. The force that produces this shearing is the resultant of all the vertical forces on one side of the section. Thus, in the simple beam of the first diagram this resultant is the reaction minus the weight of the beam between the reaction and the section; in the cantilever beam

of the second diagram it is the loads and the weight of the beam on the left of the section.

'Vertical shear' is the name given to the algebraic sum of all the vertical forces on the left of the section under consideration. Thus in the first diagram, if the

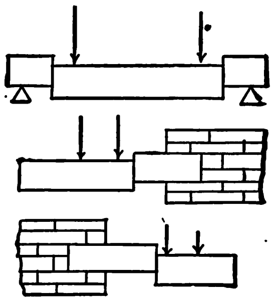

FIG. 18.

reaction be 6 000 pounds, the vertical shear V just at the right of the support is 6 000 pounds. if the beam weigh 100 pounds per linear foot, the vertical shear at a section one foot from the support and on the left of the single load is 5 900 pounds. Again in the second diagram, if the beam weigh 100 pounds per linear foot and if each concentrated load be 800 pounds, and the distance from the end to the section shown be 4 feet, the vertical shear in that section is 2 000 pounds.

It is seen from these illustrations that in a simple beam the greatest vertical shear is at the supports, and that in a cantilever beam it is at the wall. Only these sections, then, need be investigated in a solid beam. For a simple beam of length l and carrying w pounds per linear unit, the greatest vertical shear is the reaction $\frac{1}{2}wl$. For a cantilever beam of length l,

the greatest vertical shear due to uniform load is the total weight wl.

The vertical shear V produces in the cross-section an equal shearing stress. If A be the section area and S the shearing unit-stress acting over that area, then

$$V = AS, \quad S = \frac{V}{A}, \quad A = \frac{V}{S}, \tag{3}$$

are the equations similar to (1) of Art. 1: these are used for the practical computations regarding shear in solid beams.

For example, consider a steel **I** beam weighing 250 pounds per yard and 12 feet long, over which roll three locomotive wheels 4 feet apart and each bearing 14 000 pounds. The greatest shear will occur when

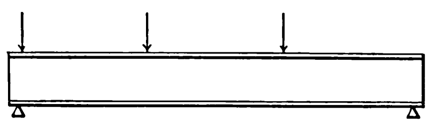

FIG 19.

one wheel is almost at the support as shown in the figure. By Art. 18 the reaction is found to be 28 500 pounds, and this is the greatest vertical shear V. By Art. 8 the area of the cross-section is found to be 24.5 square inches. Then the shearing unit-stress in the section is

$$S = \frac{28\,500}{24.5} = 1\,160 \text{ pounds per square inch.}$$

which is a low working unit-stress for steel.

As a second example, consider a wooden cantilever beam which projects out from a bridge floor and sup-

ports a sidewalk. Let it be 6 inches wide, 8 inches deep, and 7 feet long, and let the maximum load that comes upon it be 7 500 pounds. The vertical shear at the section where it begins to project is then 7 590 pounds, or the load that it carries plus its own weight. As the section area is 48 square inches, the shearing unit-stress is a little less than 160 pounds per square inch. The factor of safety against shearing is hence about 19 (Art. 6), so that the security is ample.

It is indeed only in rare instances that solid beams of uniform cross-section are subject to dangerous stresses from shearing. Beams almost universally fail by tearing apart under the horizontal tensile stresses, and hence the following articles will be devoted entirely to the consideration of these bending stresses.

Prob. 24. A simple beam of cast iron is 3×3 inches in section and $5\frac{1}{2}$ feet long between supports. Besides its own weight it is to carry a load of 4 000 pounds at the middle and a load of 1 000 pounds at $2\frac{1}{2}$ feet from the left end. Find the factor of safety against shearing.

ART. 25. RESISTANCE TO BENDING.

In Art. 24 it was shown that the resisting moment of the internal stresses in any section is equal to the bending moment of the external forces on each side of the section. Art. 19 explains how to find the bending moment, which hereafter will be designated by the letter M. In Art. 20 an expression for the resisting moment is derived. Therefore

$$\frac{SI}{c} = M \qquad (4)$$

is the fundamental formula for the discussion of the

flexure of beams, provided that the elastic limit is not exceeded. Here S is the unit-stress of tension or compression on the top or bottom of the beam, c is the vertical distance of S from the center of gravity of the cross-section, and I is the moment of inertia of the cross-section. Art. 21 explains how to find c, and Art. 22 shows how I is determined.

This formula shows that S varies directly with M, that is, the greatest tensile or compressive stress in the beam occurs at the section where M has its maximum value. For a simple beam under uniform load the bending moment M at any section distant x from the left support is, as shown in Art. 19,

$$M = \tfrac{1}{2}wl \,.\, x - wx \,.\, \tfrac{1}{2}x = \tfrac{1}{2}w(lx - x^2),$$

and if $x = \frac{1}{2}l$, this gives $M = \frac{1}{8}wl^2$ as the maximum bending moment; or if W be the total load wl, this may be written as $M = \frac{1}{8}Wl$. When concentrated loads are on a simple beam the maximum bending mo-

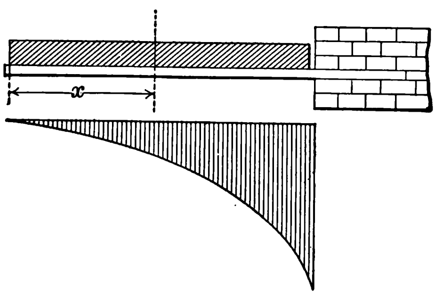

FIG. 20.

ment must usually be found by trial; it will generally be under one of those loads.

For a cantilever beam of length l the maximum bending moment always occurs at the wall. For a

uniform load of w per linear unit the bending moment at a section distant x from the end is the load wx into its lever-arm $\frac{1}{2}x$, and this is negative as it tends to produce rotation in a direction opposite to that of the hands of a watch (Art. 17). Thus, for any section $M = -\frac{1}{2}wx^2$, and when x becomes equal to l the maximum value is $M = -\frac{1}{2}wl^2$. The negative sign shows merely the direction in which rotation tends to occur, and when using formula (4) the value of M is to be inserted without sign. The diagram of bending moments for this case is a parabola, since M increases as the square of x.

For concentrated loads on a cantilever beam the bending moment M is $-P_1x$ until x passes beyond the second load; then $M = -P_1x - P_2(x - a)$ where a is the distance between the two loads. Thus the dia-

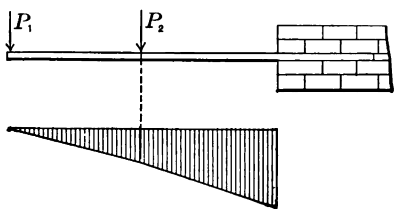

FIG. 21.

gram of bending moments is composed of straight lines, and the maximum value of M occurs when x becomes equal to l.

It may be noted that the only difference in stating moment equations for a cantilever beam and for a simple beam lies in the fact that for the former there is no reaction at the left end. A cantilever beam is hence really simpler than a simple beam, as no reactions need be computed.

are also proportional to their distances from the neutral surface.

From the above it can be shown that the neutral axis passes through the center of gravity of the cross-section. For if S be the unit-stress on the remotest fiber and c its distance from the neutral axis, then the unit-stress at the distance z is $S\frac{z}{c}$, and the total stress on an elementary area a is $S\frac{az}{c}$. The algebraic sum of all the horizontal stresses in the section then is $\frac{S}{c}\Sigma az$. From the above statical principles this sum must be zero, and it hence follows that Σaz must be zero; that is, the sum of the moments of the elementary areas is zero with respect to the neutral axis. Hence the neutral axis passes through the center of gravity, for the center of gravity is that point upon which the surface can be balanced or for which $\Sigma az = 0$.

Prob. 23. An **I** beam which is 20 feet long weighs 700 pounds and the area of its cross-section is 10.29 square inches. What is the kind of material?

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When a beam is short it sometimes fails by shearing in a vertical section near one of the supports. The force that produces this shearing is the resultant of all the vertical forces on one side of the section. Thus, in the simple beam of the first diagram this resultant is the reaction minus the weight of the beam between the reaction and the section; in the cantilever beam

each reaction is $\frac{1}{2}wl$, and the maximum bending moment M is $\frac{1}{8}wl^2$. The value of c is 2 inches, and that of I is 16 inches^4. Then since l is 36 inches,

$$\frac{SI}{c} = \frac{2\,000 \times 16}{2} = 162w,$$

from which $w = 98.8$ pounds per linear inch, and hence the total uniform safe load that can be put on the beam is about 3 560 pounds.

The student should notice that in using formula (4) all lengths must be expressed in the same unit. If the length of a beam is given in feet it must be reduced to inches for use in the formula, because S, I, and c are expressed in terms of inches. Formula (4) cannot be used to find the load that will rupture a beam, except in the manner indicated in Art. 63.

Prob. 26. A steel I beam 7 inches deep and weighing 22 pounds per foot has for the moment of inertia of its cross-section 52.5 inches^4, and it is to be used as a simple beam with a span of 18 feet. What load P can it carry when the greatest unit-stress S is required to be 12 000 pounds per square inch. (Ans. 2 930 pounds.)

ART. 27. INVESTIGATION OF BEAMS.

To investigate a beam acted upon by given loads the greatest unit-stress S produced by those loads is to be found from formula (4). From the given dimensions of the beam I and c are known, from the given loads the maximum value of M is to be found; hence

$$S = \frac{Mc}{I}$$

is the equation for computing the value of S. Then by the rules of Chapter I the degree of security of the beam is to be inferred. As formula (4) is deduced under the laws of elasticity, it fails to give reliable values of S when the elastic limit is exceeded.

For example, consider a cast-iron ⊔ section which is used as a simple beam with a span of 6 feet, and upon which there is a total uniform load of 80 000 pounds. Let the total depth be 16 inches, the total width 12 inches, the thickness of the flange 2 inches, and the thickness of the webs 1 inch. By Art. 25 the greatest value of M is at the middle of the beam, this being $\frac{1}{8} \times 80\,000 \times 6 \times 12 = 720\,000$ pound-inches. By Art. 21 the value of c is found to be 10.7 inches. By Art. 22 the value of I is found to be 1 292 inches^4. Then,

$$S = \frac{720\,000 \times 10.7}{1\,292} = 5\,960 \text{ pounds per square inch.}$$

This is the compressive unit-stress in the end of the web when the beam is placed in the ⊔ position, as is usually the case in buildings. On the base of the beam the tensile unit-stress is about half this value, since c_1 is about one-half of c. Thus under the compressive stress the beam has a factor of safety of about 15, and under the tensile stress it has a factor of safety of about 7. As the least factor of safety for cast iron should be 10, the beam has not the full degree of security required by the best practice.

Prob. 27. A piece of wooden scantling 2 inches square and 18 feet long is hung horizontally by a rope at each end and a student weighing 175 pounds stands upon it. Is is safe?

ART. 28. DESIGN OF BEAMS.

The design of a beam consists in determining its size when the loads and its length are given. The allowable working unit-stress S is first assumed according to the requirements of practice. From the given loads the maximum bending moment M is then computed. Thus in formula (4) everything is known except I and c, and

$$\frac{I}{c}=\frac{M}{S}$$

is an equation which must be satisfied by the dimensions to be selected.

For a rectangular beam of breadth b and depth d the value of c is $\frac{1}{2}d$, and the value of I is $\frac{1}{12}bd^3$. Thus the equation becomes

$$bd^2=\frac{6M}{S},$$

and if either b or d be assumed the other can be computed. For example, let it be required to design a rectangular wooden beam for a total uniform load of 80 pounds, the beam to be used as a cantilever with a length of 6 feet, and the working value of S to be 800 pounds per square inch. Here the maximum value of M is $80 \times 3 = 240$ pound-feet $= 2\,880$ pound-inches. Thus $bd^2 = 21.6$ inches^3. If b be taken as 1 inch, $d = 4.65$ inches; if b be 2 inches, $d = 3.29$ inches; if b be 3 inches, $d = 2.68$ inches. With due regard to sizes readily found in the market 3×3 inches are perhaps good proportions to adopt.

Prob. 28. A simple cast-iron beam of 14 feet span car-

ries a load of 10 000 pounds at the middle. If its width is 4 inches find its depth for a factor of safety of 10.

(Ans. 18 inches.)

ART. 29. COMPARATIVE STRENGTHS.

The strength of a beam is measured by the load it can carry with a given unit-stress S. Let it be required to investigate the relative strengths of the four following cases:

1st. A cantilever loaded at the end with W.
2d. A cantilever loaded uniformly with W.
3d. A simple beam loaded at the middle with W.
4th. A simple beam loaded uniformly with W.

Let l be the length in each case, and the cross-section be of breadth b and depth d. Then $c = \frac{1}{2}d$, and $I = \frac{1}{12}bd^3$. Then from Art. 25, and formula (4),

$$\text{For 1st,} \quad M = Wl, \quad \text{and} \quad W = \frac{Sbd^2}{6l};$$

$$\text{For 2d,} \quad M = \tfrac{1}{2}Wl, \quad \text{and} \quad W = 2\frac{Sbd^2}{6l};$$

$$\text{For 3d,} \quad M = \tfrac{1}{4}Wl, \quad \text{and} \quad W = 4\frac{Sbd^2}{6l};$$

$$\text{For 4th,} \quad M = \tfrac{1}{8}Wl, \quad \text{and} \quad W = 8\frac{Sbd^2}{6l}.$$

Hence the comparative strengths of the four cases are as the numbers 1, 2, 4, 8; that is, if four such beams are of equal size and length and of the same material, the second is twice as strong as the first, the third is four times as strong, and the last is eight times as strong as the first.

From these equations the following important laws regarding rectangular beams are derived:

> The strength varies directly as the breadth and directly as the square of the depth.
>
> The strength varies inversely as the length.
>
> A beam is twice as strong under a distributed load as under an equal concentrated load.

The second and third of these laws apply also to beams having cross-sections of any shape.

The reason why rectangular beams are placed with the longest dimension vertical is now seen to be that the strength increases in a faster ratio with the depth than with the breadth. If the breadth be doubled the strength is doubled; if the depth be doubled the strength is four times as great as before.

Prob. 29. Show that a beam 3 inches wide, 6 inches deep, and 4 feet long, is nine times as strong as a beam 2 inches wide, 4 inches deep, and $10\frac{2}{3}$ feet long.

ART. 30. STEEL I BEAMS.

Wrought-iron rolled beams have been much used in bridge and building constructions, but now mild-steel beams are almost exclusively employed. The ultimate tensile strength of such steel will be taken as 65 000 pounds per square inch, and its elastic limit as 35 000 pounds per square inch, in the solution of examples and problems hereafter given. These beams are manufactured in about thirteen different depths, and of each depth there are several different sizes or weights, so that designers have a large variety from which to select. In the following table only the heaviest and lightest sections of each depth are given. The

TABLE VIII. STEEL I BEAMS.

Depth. Inches.	Weight per Foot. Pounds.	Section Area. A Sq. Inches.	Moment of Inertia. I Inches⁴.	Section Modulus. $\frac{I}{c}$ Inches³.	Moment of Inertia. I' Inches⁴.
24	100	29.4	2 380	198	48.6
24	80	23.5	2 088	174	42.9
20	75	22.1	1 269	127	30 2
20	65	19.1	1 170	117	27.9
18	70	20.6	921	102	24.6
18	55	15.9	796	88.4	21.2
15	55	15.9	511	68.1	17.1
15	42	12.5	442	58.9	14.6
12	35	10.3	228	38.0	10.1
12	31½	9.3	216	36.0	9.50
10	40	11.8	159	31.7	9.50
10	25	7.4	122	24.4	6.89
9	35	10.3	112	24.8	7.31
9	21	6.3	85.0	18.9	5.16
8	25½	7.50	68.4	17.1	4.75
8	18	5.33	56.9	14.2	3.78
7	20	5.88	42.2	12.1	3.24
7	15	4.42	36.2	10.4	2.67
6	17¼	5.07	26.2	8.73	2.36
6	12¼	3.61	21.8	7.27	1.85
5	14¾	4.34	15.2	6.08	1.70
5	9¾	2.87	12.1	4.84	1.23
4	10½	3.09	7.1	3.55	1.01
4	7½	2.21	6.0	3.00	0.77
3	7½	2.21	2.9	1.93	0.60
3	5½	1.63	2.5	1.71	0.46

proportions of one of the sizes of the two 6-inch beams are shown in the accompanying figure, the outer line

on the right-hand side indicating the heavier section and the other one the lighter section.

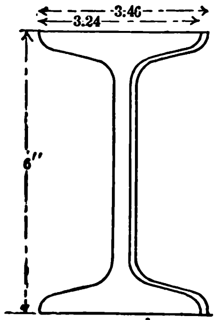

FIG. 22.

In this table the moments of inertia I in the fourth column are those about an axis through the centers of gravity and perpendicular to the web, and are those to be used in all beam computations. The values I' given in the last column are with respect to an axis through the center of gravity but parallel to the web; these are for use in the next chapter in the discussion of struts.

The quantity $\frac{I}{c}$ is often called the 'section modulus' as it contains all the dimensions of the cross-section. The process of selecting an **I** section depends merely on finding a value of $\frac{I}{c}$ which corresponds to the value of $\frac{M}{S}$, as shown in Art. 28; hence for convenience these values are tabulated in the fifth column of the table.

For example, an **I** beam in a floor is to have 20 feet span and to carry a uniform load of 13 500 pounds; what size is to be selected? The bending moment is

$M = \frac{1}{8} \times 13\,500 \times 20 \times 12 = 405\,000$ pound-inches; and the working unit-stress S should be $\frac{1}{5} \times 65\,000$ pounds per square inch. Then from formula (4),

$$\frac{I}{c} = \frac{405\,000}{13\,000} = 31.2 \text{ inches}^3,$$

and hence, from the table, the heavy 10-inch beam should be used.

Prob. 30. A heavy 15-inch steel **I** beam of 12 feet span carries a uniform load of 42 net tons. Find its factor of safety if the span be 6 feet; also if the span be 9 feet.

(Ans. 5.9 and 3.9.)

ART. 31. BEAMS OF UNIFORM STRENGTH.

The beams thus far discussed have been of uniform section throughout their entire length. As the bending moments are small near the ends of the beam the unit-stress S is there also small, and hence more material is used than is really needed. A beam of uniform strength is one so shaped that the unit-stress S is the same at all parts of the length. Such beams are sometimes made in cast iron, but rarely in other materials.

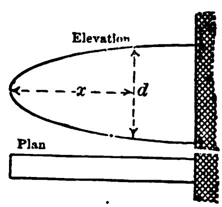

FIG. 23.

For a cantilever beam loaded with P at the end, the bending moment at any distance x from the end is Px. If the section is rectangular, formula (4) reduces

to $\frac{1}{6}Sbd^2 = Px$, in which P and S are constant. If b is made the same throughout, then

$$d^2 = \frac{6Px}{Sb},$$

and therefore d^2 must vary directly as x. If $x = l$, the value of d is the depth d_1 at the wall, and accordingly $6P/Sb = d_1^2/l$; hence the equation becomes

$$d = d_1\sqrt{\frac{x}{l}}.$$

Thus, if $x = \frac{1}{9}l$, $d = \frac{1}{3}d_1$; if $x = \frac{1}{4}l$, $d = \frac{1}{2}d_1$, and so on. As the squares of the depths vary with the distances from the end, the curve of the beam should be the common parabola.

For a rectangular cantilever beam uniformly loaded with w per linear unit the bending moment M is $\frac{1}{2}wx^2$

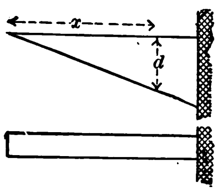

FIG. 24.

and formula (4) becomes $\frac{1}{6}Sbd^2 = \frac{1}{2}wx^2$. If the breadth be uniform throughout, then

$$d^2 = \frac{3wx^2}{Sb}.$$

Here if $x = l$, the value of d is the depth d_1 at the wall, and thus $3w/Sb = d_1^2/l^2$. Accordingly,

$$d = d_1\frac{x}{l}$$

gives the depth for any value of x, and it shows that the elevation of the beam should be a triangle.

The vertical shear near the end of the beam modifies slightly the form near the end. Thus for the first case above, if S' be the working shearing unit-stress there must be a section at the end whose area A is equal at least to P/S'.

Prob. 31. A simple beam of uniform strength is to be designed to carry a heavy load P at the middle. If d_1 be the depth at the middle, show that the depths at distances $0.1l$, $0.2l$, $0.3l$, and $0.4l$ should be $0.45d_1$, $0.63d_1$, $0.77d_1$, and $0.89d_1$.

CHAPTER V.

COLUMNS OR STRUTS.

ART. 32. GENERAL PRINCIPLES.

A bar under compression whose length is greater than about ten times its thickness is called a column or a strut. For shorter lengths the case is one of direct compression where the rules of Art. 5 apply. For the short specimen failure occurs by the shearing or splintering of the material. For the strut or column, however, failure generally occurs by a sidewise bending; this induces bending stresses, so that the phenomena of stress are more complex than in a beam.

Wooden and cast-iron columns are usually square or round, and are sometimes built hollow. Wrought-iron columns are made by riveting together channels, plates, and angle-irons. It is clear that a square or round section is preferable to a rectangular one, since then the tendency to bend is the same in all directions. For a rectangular section the bending will evidently occur in a plane parallel to the shorter side of the rectangle; thus in investigating such a column the depth d is this shorter side instead of the longer one, as in beams. When a single I beam is used as a column it tends to bend in a plane parallel to the flanges, and hence the moment of inertia to be used in its discussion is I', which is given in the last column of the table in Art. 30, the axis for this coinciding with the middle line of the web.

If a short prism whose section area is A be loaded with the weight P, the unit-stress is P/A, and this is uniformly distributed over the area A. For a column, however, this is not the case; while the mean unit-stress is still P/A the unit-stress on the concave side, if bending occurs, may be very much greater than P/A. The longer the column the greater is this unit-stress on the concave side liable to become, and hence a long column cannot carry so large a load as a short one.

There are three ways of arranging the ends of columns. Class (a) includes those with 'round ends' or

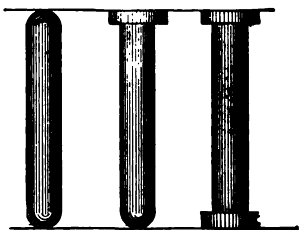

FIG. 25.

those having their ends hinged on pins. Class (b) includes those with one end round and the other fixed; the piston-rod of a steam-engine is of this type. Class (c) includes those having fixed ends; these are used in bridge and building constructions. The figure here given is a symbolical representation, and is not intended to imply that the ends of the columns are necessarily enlarged in practice. It is found by experiment that class (c) is stronger than (b), and that (b) is stronger than (a).

Prob. 32. In a certain test wrought-iron tubes 2.37 inches in outer diameter and having a section area of 1.08

square inches were used. A tube 8 feet long failed under 24 800 pounds and a tube $3\frac{1}{2}$ feet long failed under 38 200 pounds. What load would be required to cause failure for a tube only 6 or 8 inches long? (Ans. 59 000 pounds.)

ART. 33. RADIUS OF GYRATION.

In the discussion of columns a quantity called 'radius of gyration' of the cross-section is frequently used. The radius of gyration is defined to be that quantity whose square is equal to the moment of inertia of the cross-section divided by its area, or

$$r^2 = \frac{I}{A},$$

is the expression by which r^2 is to be computed. The student should observe that r has really no connection with gyration, as I has no connection with inertia, in the case of sections of beams and columns. Radius of gyration is merely a technical name, which has unfortunately come into use, to denote the square root of the quantity I/A.

The values of the radius of gyration of sections are readily obtained from the moments of inertia given in Art. 22. Thus,

For a rectangle, $r^2 = \frac{1}{12}d^2$;

For a hollow rectangle, $r^2 = \dfrac{bd^3 - b_1d_1^3}{12(bd - b_1d_1)}$.

For any I beam the value of r is at once found from the table in Art. 30; thus, for the heavy 10-inch beam, $r^2 = 159/11.8 = 13.5$ inches^2 and $r = 3.67$ inches.

As circular cross-sections are frequently used for columns, the values of the moment of inertia for these will here be stated without proof. Let d be the outer diameter and d_1 the inner diameter. Then

For a solid circle, $I = \frac{1}{64}\pi d^4$, $\qquad r^2 = \frac{1}{16}d^2$.

For a hollow circle, $I = \frac{1}{64}\pi(d^4 - d_1^4)$, $\qquad r^2 = \frac{1}{16}(d^2 + d_1^2)$.

Here the values of r^2 are found by dividing the first I by $\frac{1}{4}\pi d^2$ and the second I by $\frac{1}{4}\pi(d^2 - d_1^2)$, these being the areas of the cross-sections.

Prob. 33. Prove that the square of the radius of gyration for a hollow square section is $\frac{1}{12}(d^2 + d_1^2)$.

ART. 34. FORMULA FOR COLUMNS.

Columns and struts generally fail under the stresses produced by combined compression and bending. The phenomena are so complex that no purely theoretical formula will fully represent all cases. The formula of Rankine is that which has the best rational basis, but this cannot here be fully developed, as the laws of deflection have not yet been discussed.

Let P be the load on the vertical column, and let a horizontal plane ab cut it at the middle. If A is the section area, the average compressive unit-stress P/A may be represented by the line cd. But in consequence of the bending this is increased to aq on the concave side and decreased to bq on the convex side. The triangles pdq and qdp represent the longitudinal bending stresses, as in beams. Let the maximum unit-stress aq be denoted by S. The part ap is equal

to cd or to P/A. The part pq is due to the bending and will be denoted by S_1. Hence the maximum unit-stress aq is given by

$$S = \frac{P}{A} + S_1.$$

Now from the formula (4) established for cases of bending in Art. 25, the value of S_1 is Mc/I, where M is the bending moment of the external forces. Here

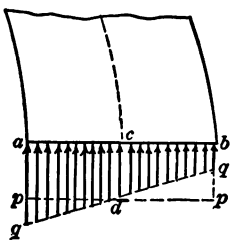

FIG. 26.

the only external force is P, and its lever arm is the lateral deflection of the central line of the column. Let this lateral deflection be called f; then $M = Pf$, and accordingly,

$$S = \frac{P}{A} + \frac{Pcf}{I},$$

where c represents the distance ac in the figure.

Now let I be replaced by Ar^2, where r is the radius of gyration of the cross-section. Then the preceding equation becomes

$$S = \frac{P}{A}\left(1 + \frac{cf}{r^2}\right),$$

which shows how the unit-stress S on the concave side

increases with the lateral deflection f. By a discussion of the subject of deflection, such as is given in 'Mechanics of Materials,' it is shown that the value of

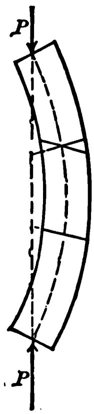

FIG. 27.

f which is liable to occur increases as the square of the length of the beam, or cf may be made equal to ql^2, where q depends upon the kind of material and the arrangement of the ends (see Art. 48). The last equation may now be written,

$$\frac{P}{A} = \frac{S}{1 + q\frac{l^2}{r^2}}, \qquad (5)$$

which is Rankine's formula for columns.

The values of q to be used in problems and examples in this book are given in the following table. These mean values have been derived by the consideration of numerous experiments on the rupture of

TABLE IX. COLUMN CONSTANTS q.

Material.	Both Ends Fixed.	One Fixed End and One Round End.	Both Ends Round.
Timber	$\frac{1}{3\,000}$	$\frac{1.78}{3\,000}$	$\frac{4}{3\,000}$
Cast Iron	$\frac{1}{5\,000}$	$\frac{1.78}{5\,000}$	$\frac{4}{5\,000}$
Wrought Iron	$\frac{1}{35\,000}$	$\frac{1.78}{35\,000}$	$\frac{4}{35\,000}$
Steel	$\frac{1}{25\,000}$	$\frac{1.78}{25\,000}$	$\frac{4}{25\,000}$

columns and struts. It is seen that in all cases q is four times as large for round ends as for fixed ends, which results from the fact that a very long column with round ends has only one-fourth the strength of one with fixed ends.

Prob. 34. If $P/A = 500$ pounds per square inch for a timber column with fixed ends, find from (5) the values of S when $l/r = 0$, $l/r = 50$, and $l/r = 100$.

ART. 35. SAFE LOADS FOR COLUMNS.

To find a safe load for a column of given size and material the working value of S is to be assumed by Art. 7. The value of r is determined by Art. 33, and q by the table in Art. 34. Then, from the formula (5),

$$P = \frac{AS}{1 + q\frac{l^2}{r^2}},$$

which gives the safe load for the column.

For example, let it be required to find the safe load for a timber strut 3×3 inches and 5 feet long, so that the greatest compressive unit-stress S may be 800 pounds per square inch. Here $b = d = 3$ inches, $r^2 = \frac{1}{12}d^2 = \frac{3}{4}$ inches², $l^2 = 3\,600$ inches², $l^2/r^2 = 4\,800$, $q = 1/3\,000$, $ql^2/r^2 = 1.6$. Then

$$P = \frac{9 \times 800}{1 + 1.6} = 2\,770 \text{ pounds,}$$

which is the safe load for the strut. If the length be only about one foot, the safe load will be simply $P = 9 \times 800 = 7\,200$ pounds. If the length be 12 feet, P will be found by the formula to be only 700 pounds. The influence of the length on the safe load is hence very great.

Prob. 35. A hollow cast-iron column to be used in a building is 6×6 inches outside dimensions and 5×5 inches inside dimensions, the length being 18 feet, and the ends fixed. Find its safe load. (Ans. 58 000 pounds.)

ART. 36. INVESTIGATION OF COLUMNS.

The investigation of a column under a given load consists in computing the unit-stress S from formula (5) and then comparing this with the ultimate strength and elastic limit of the material, having due regard to whether the stresses are steady, variable, or sudden (Art. 7). The value of S is

$$S = \frac{P}{A}\left(1 + q\frac{l^2}{r^2}\right),$$

and the given data will include all the quantities in the second member.

For example, a wrought-iron tube used as a column with fixed ends carries a load of 38 000 pounds. Its outside diameter is 6.36 inches, its inside diameter 6.02 inches, and its length 18 feet. It is required to find the unit-stress S and the factor of safety. Here $P = 38\,000$ pounds, $A = \frac{1}{4}\pi(6.36^2 - 6.02^2) = 3.31$ square inches, $q = 1/35\,000$, $l = 18 \times 12 = 216$ inches, $r^2 = \frac{1}{16}(6.36^2 + 6.02^2) = 4.79$ inches^2. Then by the formula

$$S = \frac{38\,000}{3.31}\left(1 + \frac{216 \times 216}{35\,000 \times 4.79}\right),$$

or $S = 14\,700$ pounds per square inch. The factor of safety is thus about 4, which is a safe value if the column is used under steady stress, but too small if sudden stresses or shocks are liable to occur. If the length of this column be 36 feet, the unit-stress S will become about 25 000 pounds per square inch, so that its factor of safety will be only 2.2, a value far too low for proper security.

As a second example let a heavy 10-inch steel I beam which is 25 feet long be used as a strut in a bridge truss, the ends being hinged on pins. Let the compression on it be 5 900 pounds. Here from the table in Art. 30 $A = 11.8$ square inches and $I' = 9.50$ inches^4, whence $r^2 = 0.80$ inches^2; also $q = 4/25\,000$, $l = 300$ inches, $P = 5\,900$ pounds. Then from the formula S is found to be 9 500 pounds per square inch, which is about one-third of the elastic limit of the material, and hence a safe value.

Prob. 36. A pine stick 3×3 inches and 12 feet long is used as a column with fixed ends. Find its factor of safety under a load of 3 000 pounds. If its length be only one foot, what is the factor of safety?

ART. 37. DESIGN OF COLUMNS.

When the length of a column is given and the load to be carried by it, the design consists in selecting the proper material and then finding the dimensions so that S may have the proper working value. This in general must be done by trial, dimensions being assumed and inserted in formula (5), and if these do not fit changes are to be made in them until a satisfactory agreement is found.

For example, let it be required to find the size of a square wooden column with fixed ends and 24 feet long to carry a load of 100 000 pounds with a factor of safety of 10. Here as the ultimate strength of timber is 8 000 pounds per square inch the working value of S is 800 pounds per square inch. If the column be very short the area A will be 125 square inches and the side of the square about 11 inches, hence for the actual case the side of the square is more than 11 inches. Assume it 18 inches, and then from the formula of the last article find the value of S; this being less than 800, shows that 18 inches is too great. Again, trying 16 inches, this will be found to give a value of S a little larger than 800, while 17 inches makes S a little smaller than 800. Thus, $16\frac{1}{2}$ inches is a satisfactory solution of the problem.

Prob. 37. Find what steel I beam 12 feet long may be used as a column to carry a load of 100 000 pounds, taking the working value of S at 12 000 pounds per square inch.

CHAPTER VI.

THE TORSION OF SHAFTS.

ART. 38. PHENOMENA OF TORSION.

Torsion is that kind of stress which occurs when external forces tend to twist a body round an axis. A shaft which transmits power is twisted by the forces applied to the pulleys, and thus all its cross-sections are brought into stress. This stress is a kind of shearing, but the forces acting in different parts of a section are not parallel.

Let one end of a horizontal bar be rigidly fixed, and to the free end let a lever be attached at right angles

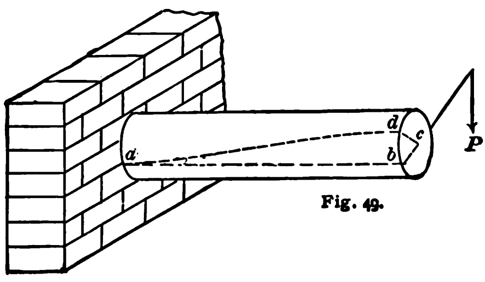

FIG. 28.

to its axis. A weight P hung at the end of this lever will twist the shaft so that a line ab which originally was horizontal will assume a spiral form ad, while the radial line cb will move to the position cd. It has been shown by experiments that, if the material is not

stressed beyond its elastic limit, the angles *bcd* and *bad* are proportional to the applied weight P, and that on the removal of this weight the lines *cd* and *ad* will return to their original position. If the elastic limit be exceeded this proportionality does not hold, and if the stress be made great enough the bar will be ruptured.

Let p be the lever-arm of P with respect to the axis c. Then experience also shows that the amount of twist is proportional to p. The product Pp is the moment of P with respect to the axis, and it is called the 'twisting moment.' If there be several forces P_1, P_2, etc., acting on the shaft with lever-arms p_1, p_2, etc., the total twisting moment Pp is the algebraic sum of the separate moments P_1p_1, P_2p_2, etc., those being positive which tend to turn in the direction of the hands of a watch, and those negative which turn in the opposite direction.

For example, let the three lever-arms be applied to a bar at the points B, C, and D, whose distances from A

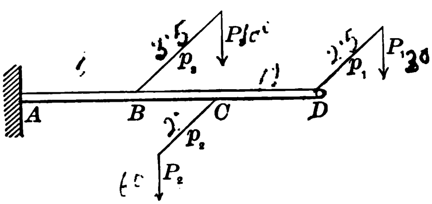

FIG. 29.

are 5, 8, and 12 feet. Let the forces in the figure be $P_1 = 30$ pounds, $P_2 = 60$ pounds, and $P_3 = 100$ pounds, their lever-arms being $p_1 = 2.5$ feet, $p_2 = 2.0$ feet, and $p_3 = 3.5$ feet. Then for all sections between D and C the twisting moment is $+ 30 \times 2.5 = + 75$ pound-feet; for all sections between C and B the twisting mo-

ment is $+ 30 \times 2.5 - 60 \times 2.0 = - 105$ pound-feet; and for all sections between B and A the twisting moment is $+ 30 \times 2.5 - 60 \times 2.0 + 100 \times 3.5 = + 245$ pound-feet. Thus the tendency to twist between B and C is in the opposite direction to that in the other parts of the bar.

Prob. 38. If a force of 600 pounds acting at 5 inches from the axis twists the end of a shaft 30 degrees, what force acting at 12 inches from the axis will twist it 60 degrees. (Ans. 50 pounds.)

ART. 39. POLAR MOMENTS OF INERTIA.

In the discussion of shafts the moments of inertia of cross-sections are required with respect to a point at the center of the shaft and not with respect to an axis in the same plane, as in beams and columns. The 'polar moment of inertia' of a surface is defined as the sum of the products obtained by multiplying each elementary area by the square of its distance from the center of the surface. Thus if a be any elementary area and x its distance from the center the quantity Σax^2 is the polar moment of inertia.

In the figure let a be any elementary area and z its distance from an axis AB passing through the center

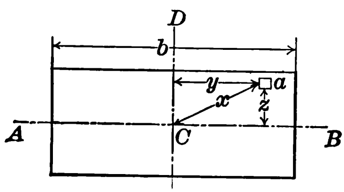

FIG. 30.

of gravity of the section; then Σaz^2 is the moment of inertia with respect to the axis AB (Art. 22). Also, if

y is the distance from a to an axis CD which is normal to AB, then Σay^2 is the moment of inertia with respect to the axis CD. But since $z^2 + y^2 = x^2$, the product Σax^2 is equal to $\Sigma az^2 + \Sigma ag^2$; that is, the polar moment of inertia is the sum of the moments of inertia taken with respect to any two rectangular axes.

The polar moment of inertia is represented by J. By the aid of the above principle its value is readily found from the values of I given in Arts. 22 and 33. Let d be the diameter of a circle or the side of a square; then

For a solid circle, $J = \frac{1}{32}\pi d^4$;

For a solid square, $J = \frac{1}{6}d^4$.

Also, in the case of a hollow section let d_1 be the inner diameter or side; then,

For a hollow circle, $J = \frac{1}{32}\pi(d^4 - d_1^4)$;

For a hollow square, $J = \frac{1}{6}(d^4 - d_1^4)$.

The circular sections are most frequently used for shafts, square ones being of infrequent occurrence.

Prob. 39. Show that the polar moment of inertia for the rectangle in the above figure is $\frac{1}{6}bd(b^2 + d^2)$.

ART. 40. FORMULA FOR TORSION.

If two cross-sections be taken in a shaft very near together, each section tends to twist with respect to the other, and shearing stresses are found to exist in all parts of the section. These shearing stresses are least at the center and greatest at the boundary of the sec-

tion, and they act everywhere perpendicular to the lever-arms drawn to them from the center. If the elastic limit be not exceeded it is found that the stresses are proportional to their lever-arms.

Let P be the force acting with the lever-arm p which produces the twisting moment Pp. This must be equal

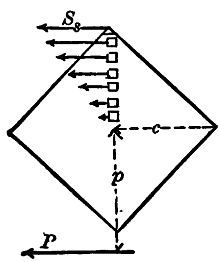

FIG. 31.

to the resisting moment of the internal stresses. Let S be the shearing unit-stress at the remotest part of the section whose distance from the center is c. Then the stress at a distance $\frac{1}{2}c$ from the center is $\frac{1}{2}S$, and the stress at a distance x from the center is $S\frac{x}{c}$. The total stress on an elementary area a at a distance x from the center is then $S\frac{ax}{c}$, and the moment of this stress with respect to the center is $S\frac{ax^2}{c}$. The resisting moment is the sum of all the values of $S\frac{ax^2}{c}$, or, since S and c are constants, this sum is $\frac{S}{c}\Sigma ax^2$. But, as seen in the last article, the quantity Σax^2 is the polar moment of iner-

tia J. Accordingly the resisting moment of the internal shearing stresses is $\frac{SJ}{c}$, and, equating this to the twisting moment, there results

$$\frac{SJ}{c} = Pp, \qquad (6)$$

which is the fundamental formula for torsion.

This formula is analogous with formula (4) for beams, and is used in a similar manner to investigate and design shafts. The unit-stress S is here always a shearing stress, and its working values are to be determined by applying factors of safety to the ultimate shearing strengths given in Art. 6. Shafts which transmit power are subject to variable loads and often to shocks, and hence their values of S should be taken low. Formula (6) is subject to the same limitation as formula (4), namely, it is only true when the unit-stress S is less than the elastic limit of the material (see Art. 63).

For example, if $Pp = 30\,000$ pound-inches, let it be required to find the shearing stress produced by it in a circular shaft 4 inches in diameter. Here $c = 2$ inches, $J = 25.13$ inches^4, and then by (6) the value of S is found to be 1 590 pounds per square inch. If the shaft be wood this is too low a value of S, it being about one-half of the ultimate shearing strength; if it be wrought iron or steel there is a high factor of safety.

Prob. 40. A round steel shaft is subject to a twisting moment of 2 500 pound-inches. What should be its diameter so that the greatest shear S may be 6 000 pounds per square inch? (Ans. 1.3 inches.)

ART. 41. SHAFTS TO TRANSMIT POWER.

'Work' is the product of a force by the distance through which it is exerted. Thus, if a weight of 10 pounds be lifted vertically a distance of 5 feet there are performed 50 foot-pounds of work. If this weight be moved horizontally, however, the force required depends only on frictional and other resistances; if these require a force of 3 pounds and this be exerted through a distance of 5 feet, then 15 foot-pounds of work are performed.

'Power' is work performed in a given time. The unit of power is the 'horse-power' which is defined as 33 000 foot-pounds of work performed in one minute. Thus, if 99 000 foot-pounds of work are performed in one minute, the power exerted is 3 horse-powers; if 99 000 foot-pounds of work be performed in two minutes, the power exerted is $1\frac{1}{2}$ horse-powers.

Power from a motor is usually transmitted to a shaft by belts, and the shaft then transmits the power to the places where the work is to be performed. In doing this the shaft is brought under stress. Let H be the power transmitted through a belt to a pulley. Let P be the tangential force in pounds brought by the belt on the circumference of the pulley, and let p be the radius of the pulley in inches. Let n be the number of revolutions made by the shaft and pulley in one minute. In one revolution P pounds is exerted through $2\pi p$ inches, and the work of $P \times 2\pi p$ pound-inches, or $\frac{1}{6}\pi Pp$ pound-feet, is performed. In one minute the work performed is $\frac{1}{6}n\pi Pp$ pound-feet. The number of

horse-powers exerted is found by dividing this work by 33 000, or

$$H = \frac{n\pi Pp}{198\,000}.$$

The twisting moment Pp may now be replaced by the resisting moment SJ/c, and hence

$$\frac{SJ}{c} = \frac{198\,000H}{n\pi}, \qquad (7)$$

which is the formula for the discussion of shafts that are used to transmit power.

For a round shaft c is equal to one-half the diameter; for a square shaft c is one-half of the diagonal. The unit-stress S is here always that for shearing, and in selecting its safe value a high factor of safety is to be used, as the shaft is subject to variable stresses. It is noticed that S varies inversely with n, that is, for a given power transmitted the slower the speed the greater is the stress in the shaft.

Prob. 41. If a round shaft one inch in diameter transmits one horse-power at 100 revolutions per minute, show that the shearing stress produced is about 3 200 pounds per square inch.

ART. 42. SQUARE SHAFTS.

Square shafts are rarely used except for wooden water-wheels. If d be the side of the square, J in formula (7) is $\frac{1}{12}d^4$, and c is the half-diagonal $\frac{1}{2}d\sqrt{2}$. Thus the formula reduces to

$$Sd^3 = 267\,500\frac{H}{n},$$

from which either S or d may be computed when the other is given.

For example, let a square wooden shaft 12 inches in size transmit 118 horse-powers at 25 revolutions per minute; what is its factor of safety? Here $H = 118$ horse-powers, $n = 25$, $d = 12$ inches, and then

$$S = \frac{267\,500 \times 118}{25 \times 1728} = 730 \text{ pounds per square inch,}$$

and hence the factor of safety against shearing is 3 000/730 or about 4; this is too low a value for timber.

Again let it be required to find the size of a square wooden shaft to transmit 118 horse-powers at 25 revolutions per minute, so that S may have the safe value of 200 pounds per square inch. Here, from the formula,

$$d^3 = \frac{267\,500 \times 118}{25 \times 200} = 6\,313 \text{ inches}^3,$$

and hence d should be 18.5 inches.

Prob. 42. Find the power that can be transmitted by a cast-iron shaft 3 inches square when making 10 revolutions per minute, the value of S not to exceed 1 200 pounds per square inch. (Ans. 1.2 horse-powers.)

ART. 43. ROUND SHAFTS.

For solid round shafts of diameter d the formula (7) reduces to the simpler form,

$$Sd^3 = 321\,000 \frac{H}{n},$$

which is used for the investigation and design of round shafts, d being always in inches and S in pounds per square inch.

For example, let it be required to design a wrought-iron shaft to transmit 90 horse-power when making 250 revolutions per minute. Here the factor of safety may be about 8, or S may be about 7 000 pounds per square inch. Then from the formula the diameter d is found to be $2\frac{5}{8}$ inches.

Hollow forged steel shafts are now much used for ocean steamers, as their strength is greater than solid shafts of the same area of cross-section. If d be the outside and d_1 the inside diameter, the value of J is $\frac{\pi}{32}(d^4 - d_1^4)$ and c is $\frac{1}{2}d$. These inserted in (7) give

$$S\frac{d^4 - d_1^4}{d} = 321\,000\frac{H}{n},$$

which is the formula for investigation and discussion.

For example, a nickel steel shaft of 17 inches outside diameter is to transmit 16 000 horse-powers at 50 revolutions per minute; what should be the inside diameter so that the unit-stress S may be 25 000 pounds per square inch? Here everything is given except d_1, and from the equation its value is found to be 11 inches nearly. The area of the cross-section of this shaft will be about 132 square inches, and its weight per linear foot about 449 pounds.

Prob. 42. If a hollow shaft have the same area of cross-section as a solid one, and if the inside diameter of the hollow shaft be one-half of the outside diameter, prove that the hollow shaft is 44 per cent stronger than the solid one.

CHAPTER VII.

ELASTIC DEFORMATIONS.

ART. 44. THE COEFFICIENT OF ELASTICITY.

It was explained in Chapter I that, when a bar is subject to stresses produced by gradually applied forces, the elongations increase proportionately with the stresses, if the elastic limit is not exceeded. This law of elasticity enables the elongations of bars and the deflections of beams to be computed, provided that none of the stresses exceeds the elastic limit of the material.

The 'coefficient of elasticity' in tension is the ratio of the unit-stress to the unit-elongation. Thus, if a bar one inch long and one square inch in cross-section be under the stress S an elongation s is produced, and

$$E = \frac{S}{s} \tag{8}$$

is the coefficient of elasticity. If the bar have a section area A which is acted on by the pull P, then the unit-stress S is P/A; if the bar have the length l an elongation e is produced and the unit-elongation s is given by e/l.

For compression E is the ratio of the unit-stress to the unit-shortening accompanying that stress, and in general E is the ratio of the unit-stress to the unit-deformation. As s is an abstract number, E is ex-

pressed in the same unit as S, that is, in pounds per square inch or kilos per square centimeter.

Within the elastic limit S increases at the same rate as s, and thus E is a constant; beyond the elastic limit there is no proper coefficient of elasticity. For different materials under the same unit-stress S, the value of E increases as s decreases; thus E is a measure of the stiffness of materials within the elastic limit. Some writers use the term 'modulus of elasticity' for E, but the term here employed is more rational, because in the equation $S = Es$ the letter E is the coefficient of the elastic deformation s.

The values of the coefficients of elasticity for tension and compression are practically the same, and their mean values for the different materials are given in the following table. For shear the coefficients of

TABLE X. COEFFICIENTS OF ELASTICITY.

Material.	Pounds per Square Inch.	Kilos per Sq. Centimeter.
Timber	1 500 000	105 000
Cast Iron	15 000 000	1 050 000
Wrought Iron	25 000 000	1 750 000
Steel	30 000 000	2 100 000

elasticity are about one-third of those stated in the table. These values show that, within the elastic limit, steel is the stiffest of the four materials, it being 20 per cent stiffer than wrought iron, twice as stiff as cast iron, and twenty times as stiff as timber. In other words, a given stress less than the elastic limit will elongate a timber bar twenty times as much as a

steel bar, a cast-iron bar twice as much, and a wrought-iron bar 20 per cent more.

Prob. 44. A bar one inch square and 2 inches long elongates 0.0004 inches under a tension of 5 000 pounds. Compute the coefficient of elasticity.

ART. 45. ELONGATION UNDER TENSION.

Let a bar whose section area is A and whose length is l be under the tension P, and let e be the elongation produced. The unit-stress S is P/A and the unit-elongation s is e/l. Then the coefficient of elasticity E is

$$E = \frac{S}{s} = \frac{Pl}{Ae};$$

and hence, if P/A be less than the elastic limit,

$$e = \frac{Pl}{AE}$$

is the elastic elongation of the bar due to the applied tension P.

For example, let it be required to find the elongation of a wrought-iron bar 30 feet long when stressed up to its elastic limit. Here $P/A = 25\,000$ pounds per square inch, $E = 25\,000\,000$ pounds per square inch, and $l = 360$ inches. Then from the formula $e = 0.36$ inches. This is the elastic elongation; the ultimate elongation will be about 72 inches. In all cases, as seen from the figure in Art. 4, the elastic elongations are very small compared with the ultimate elongations.

Prob. 45. A steel eye-bar 30 feet long is $1\frac{1}{2} \times 6$ inches in size. How much does it elongate under a pull of 90 000 pounds? (Ans. 0.12 inches.)

ART. 46. SHORTENING UNDER COMPRESSION.

If a bar of cross-section A and length l be under the compression P it shortens the amount e. For a short bar where no lateral deflection can occur the unit-stress S is uniform over the cross section, and the shortening follows the same law as does the elongation in tension, and hence

$$e = \frac{Pl}{AE}.$$

Here, as before, the unit-stress P/A must not exceed the elastic limit of the material.

For example, let a cast-iron bar one inch in diameter and 5 inches long be under a compression of 10 000 pounds. Here $P = 10\,000$ pounds, $A = 0.785$ square inches, $l = 5$ inches, and $E = 15\,000\,000$ pounds per square inch. Then from the formula $e = 0.0042$ inches; but this result is of no value, because the unit-stress P/A is nearly double the elastic limit of cast iron. If, however, P be given as 3 000 pounds, then the formula properly applies, and e is found to be 0.0013 inches.

Prob. 46. A wrought-iron bar 18 inches long weighs 24 pounds. How much will it shorten under a compression of 7 250 pounds? (Ans. 0.0011 inches.)

ART. 47. DEFLECTION OF CANTILEVER BEAMS.

The best method of deriving formulas for the deflections of beams is by the help of the calculus. These methods are given in higher works on the subject; see for instance Merriman's Mechanics of Materials. The formulas will be stated here without proof, but

be accompanied by illustrations showing their value and importance.

When a load P is at the end of a cantilever beam whose length is l, a deflection of that end results, which will be designated by f. This deflection will evidently

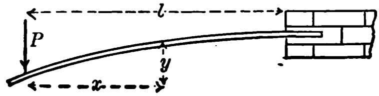

FIG. 32.

be the greater the greater the load and the longer the length of the beam. The formula for it is

$$f = \frac{Pl^3}{3EI},$$

in which E is the coefficient of elasticity of the material (Art. 44) and I is the moment of inertia of the cross-section (Art. 22).

When a uniform load is on the beam let this be called W. Then the deflection is

$$f = \frac{Wl^3}{8EI}.$$

It is thus seen that the deflection varies as the cube of the length of the beam, so that if the length be doubled

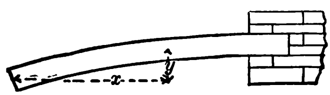

FIG. 33.

the deflection will be eight times as great. It is also seen that a uniform load produces only three-eighths as much deflection as a single load at the end.

For example, let it be required to compute the deflection of a cast-iron cantilever 2×2 inches and 6

feet long, due to a load of 100 pounds at the end. Here $P = 100$ pounds, $l = 72$ inches, $E = 15\,000\,000$ pounds per square inch, and $I = \frac{1}{12}2^4 = 1\frac{1}{3}$ inches^4. Then from the formula $f = 0.622$ inches, which is the deflection at the end.

For a rectangular cross-section of breadth b and depth d the value of I is $\frac{1}{12}bd^3$. Thus the deflections of rectangular beams vary inversely as b and d^3. As stiffness is the reverse of deflection, it is seen that the stiffness of a beam is directly as its breadth, directly as the cube of its depth, and inversely as the cube of its length. The laws of stiffness are hence quite different from those of strength.

Prob. 47. A steel **I** beam 8 inches deep and 6 feet long is used as a cantilever to carry a uniform load of 240 000 pounds. What will be its deflection?

ART. 48. DEFLECTION OF SIMPLE BEAMS.

When a simple beam of span l has a load P at the middle, each reaction is $\frac{1}{2}P$. If this beam be imag-

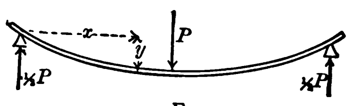

FIG. 34.

ined to be inverted it will be seen to be equivalent to two cantilevers of length $\frac{1}{2}l$, each having the load $\frac{1}{2}P$ at the end. Hence in the first formula for the deflection of a cantilever, given in Art. 47, if l be replaced by $\frac{1}{2}l$ and P by $\frac{1}{2}P$, it becomes

$$f = \frac{Pl^3}{48EI},$$

which gives the deflection of a simple beam due to a load at the middle.

When a simple beam is loaded with w per linear unit the total load wl is represented by W. The deflection at the middle due to this load is

$$f = \frac{5Wl^3}{384EI},$$

which is only five-eighths of the deflection caused by the same load at the middle.

The formulas of this and the preceding article are only valid when the greatest horizontal stress S produced by the load is less than the elastic limit. These formulas can be expressed in terms of S by substituting the values of P and W from the formula (4) of Art. 25. Thus for the simple beam with load at the middle $\frac{1}{4}Pl = SI/c$, and for the uniform load $\frac{1}{8}Wl = SI/c$. Hence,

$p = \frac{SI}{lc}$

For the single load P, $$f = \frac{Sl^2}{12Ec};$$

For the uniform load W, $$f = \frac{5Sl^2}{48Ec};$$

which show that the deflections of beams under the same unit-stresses increase directly as the squares of their lengths.

Prob. 48. In order to find the coefficient of elasticity of oak, a bar 2 × 2 inches, and 6 feet long, was loaded at the middle with 50 pounds, and then with 100 pounds, the corresponding deflections being found to be 0.16 and 0.31 inches. Compute the coefficient of elasticity E.

ART. 49. RESTRAINED BEAMS.

A beam is said to be restrained at one end when that end is horizontally fixed in a wall and the other end rests on a support. In this case the reaction of

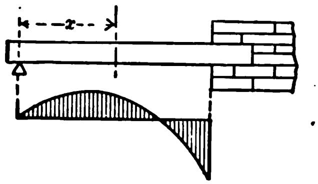

FIG. 35.

the support is less than for a simple beam. For a uniform load of w per linear unit over the span l it is proved in 'Mechanics of Materials' that the reaction at the support is $\frac{3}{8}wl$, provided the elastic limit be not exceeded. The bending moment at any section distant x from the support then is $\frac{3}{8}wlx - \frac{1}{2}wx^2$, and this shows that when $x = \frac{3}{4}l$ there is no bending moment; when $x = \frac{3}{8}l$ the greatest positive bending moment is $\frac{9}{128}wl^2$, and when $x = l$ the greatest negative bending moment is $\frac{1}{8}wl^2$; the distribution of bending moments being as shown in the figure. Also, the maximum deflection is

$$f = \frac{wl^4}{185EI} = \frac{Wl^3}{185EI},$$

which occurs when x has the value $0.4215l$.

For a beam fixed at both ends and uniformly loaded there is a negative bending moment $\frac{1}{12}wl^2$ at each wall and a positive bending moment $\frac{1}{24}wl^2$ at the middle; also the deflection at the middle is

$$f = \frac{Wl^3}{384EI},$$

in which W is the total uniform load wl.

It is seen that in these restrained beams the lower side is partly in tension and partly in compression, since a positive bending moment indicates the former and a negative one the latter (Art. 25). For a simple beam the greatest bending moment is $\frac{1}{8}Wl$, for a beam fixed at both ends the greatest bending moment is $\frac{1}{12}Wl$; hence if both be the same size the restrained

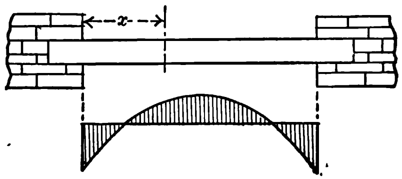

FIG. 36.

beam will carry the greater load, or if both carry the same load the restrained beam may be of smaller size than the simple one. Thus if beams can be fixed horizontally at their ends the construction may be more economical.

Prob. 49. When a beam is fixed at one end and supported at the other, the reaction of the supported end due to a load P at the middle is $\frac{5}{16}P$. Show that there is a positive bending moment $\frac{5}{32}Pl$ under the load, and a negative bending moment $\frac{3}{16}Pl$ at the wall. Also draw the diagram of bending moments.

ART. 50. TWIST IN SHAFTS.

When a shaft of length l transmits H horse-powers at a speed of n revolutions per minute, one end of the shaft is twisted with respect to the other through an angle of D degrees. If the elastic limit be not exceeded, this angle is

$$D = 3\,560\,000 \frac{Hl}{nFJ},$$

in which J is the polar moment of inertia of the section (Art. 39) and F is the coefficient of elasticity for shear (Art. 44). Here l must be taken in inches, J in inches^4, and F in pounds per square inch.

For example, let a steel shaft 125 feet long, 17 inches outside diameter, and 11 inches inside diameter transmit 16 000 horse-powers at 50 revolutions per minute. Here $H = 16\,000$ horse-powers, $l = 1\,500$ inches, $n = 50$, $F = 10\,000\,000$ pounds per square inch, $J = 6\,765$ inches^4. Then from the formula $D = 25.3$ degrees, which is the angle through which a point on one end is twisted relative to the corresponding point on the other end.

If this shaft revolve with a speed of only 25 revolutions per minute while doing the same work, its angle of twist will be twice as great and the stresses in it also twice as great as before. The formula also shows that the angle of twist varies directly as the length of the shaft.

Prob. 50. A solid steel shaft 125 feet long and 16 inches in diameter transmits 8 000 horse-powers at a speed of 25 revolutions per minute. Compute the angle of twist.

CHAPTER VIII.

RESILIENCE OF MATERIALS.

ART. 51. FUNDAMENTAL IDEAS.

When a force of uniform intensity P is exerted through the distance e the work performed is measured by the product Pe. When a bar is tested in a machine, however, the force gradually and uniformly increases from 0 up to the value P and produces the elongation e; here the work performed is $\frac{1}{2}Pe$, because the average value of the uniformly increasing force is $\frac{1}{2}P$. In the first place the work may be represented by a rectangle of height P and base e; in the second case the work may be represented by a triangle of height P and base e; and the area of the triangle is one-half that of the rectangle.

As the external force increases from 0 up to P the internal stress in the bar increases gradually and uniformly from 0 up to S. The internal work of these stresses is called the 'resilience' of the bar. As the internal work equals the external work $\frac{1}{2}Pe$, this quantity is a measure of the resilience.

Strength is the capacity of a body to resist force; stiffness is the capacity of a body to resist deformation; resilience is the capacity of a body to resist work. The higher the resilience of a material the greater is its capacity to resist the work of external forces.

Elastic resilience is that internal work which has been

performed when the internal stress reaches the elastic limit. Ultimate resilience is that internal work which has been performed when the body is ruptured. Ultimate strength is usually from two to three times the elastic strength; ultimate elongation is always much greater than elastic elongation; and ultimate resilience is very much larger than elastic resilience.

Resilience, like work, is expressed in foot-pounds, or inch-pounds, usually in the latter unit. Thus, if a bar be subject to a stress which gradually and uniformly increases from 0 up to 5 000 pounds and is accompanied by an elongation of 0.5 inches, the resilience is 1 250 pound-inches.

Prob. 51. If a wrought-iron bar weighing 30 pounds per linear foot be subject to a stress of 5 000 pounds per square inch which is accompanied by an elongation of 0.5 inches, what is the resilience? (Ans. 11 250 inch-pounds.)

ART. 52. ELASTIC RESILIENCE OF BARS.

Let a bar of length l and section area A be under a tension P, which produces a unit-stress S equal to the elastic limit of the material and an elongation e. The elastic resilience of the bar is then equal to $\frac{1}{2}Pe$. Now $P = SA$, and by Art. 45 the elastic elongation is $e = Pl/AE = Sl/E$; hence letting K represent the elastic resilience, the product $\frac{1}{2}Pe$ becomes

$$K = \frac{S^2}{2E} Al, \tag{9}$$

or the elastic resilience of a bar is proportional to its section area and to its length, that is, to its volume.

If the bar have a section of one square inch and a length of one inch, then Al is one cubic inch, and the elastic resilience is

$$k = \frac{S^2}{2E},$$

in which S is the elastic limit of the material.

This quantity is called the modulus of resilience, since for any given material it is a constant. For bars under tension the average values of S are given in Art. 2, and those of E in Art. 44. Using these constants, the 'modulus of resilience' k has the following values for tension:

For timber, $k = 3$ inch-pounds;
For cast iron, $k = 1$ inch-pound;
For wrought iron, $k = 12$ inch-pounds;
For steel, $k = 42$ inch-pounds.

These figures show that the capacity of steel to resist work within the elastic limit is the greatest of the four materials, and that of cast iron the least.

For a bar of any size the elastic resilience is found by multiplying its volume by the modulus of resilience k. Thus, a bar of timber whose volume is 50 cubic inches has an elastic resilience of about 150 inch-pounds, that is, the external work required to stress it up to the elastic limit is 150 inch-pounds. The particular shape of the bar is unimportant; it may be 5 inches in section area and 10 inches long, or 2 inches in section area and 25 inches long, or any other dimensions which give a volume of 50 cubic inches.

The above formula (8) also gives the work required to produce any unit-stress S which is less than the

elastic limit. For example, let it be required to find the work needed to stress a bar of wrought iron up to 12 500 pounds per square inch, the diameter of the bar being 2 inches and its length 18 feet. Here $S = 12\,500$ pounds per square inch, $E = 25\,000\,000$ pounds per square inch, $A = 3.14$ square inches, and $l = 216$ inches. Then

$$K = \frac{12\,500^2 \times 3.14 \times 216}{2 \times 25\,000\,000} = 2\,120 \text{ inch-pounds.}$$

If the bar is required to undergo this stress 250 times per minute, the work required in one minute is $250 \times 2\,120 = 530\,000$ inch-pounds $= 44\,200$ foot-pounds. The power expended in stressing the bar is hence $44\,200/33\,000 = 1.34$ horse-powers.

When a bar is under a unit-stress S_1 and this is increased by additional exterior loads to S_2, the resilience is

$$K = (S_2^2 - S_1^2)\frac{Al}{2E},$$

provided that S_2 be not greater than the elastic limit.

Prob. 52. A bar of steel 10 feet long and weighing 490 pounds is stressed in one second from 4 000 up to 9 000 pounds per square inch. What work and what horse-power are expended in doing this?

ART. 53. ELASTIC RESILIENCE OF BEAMS.

When a simple beam of span l is brought into stress by a load P applied gradually and uniformly at the middle, the deflection f results and the work $\frac{1}{2}Pf$ is performed. This work equals the resilience of the beam. The value of f in terms of the horizontal unit-stress is given in Art. 48, and the value of P in terms

of S is given from (4) of Art. 25. Thus the product $\frac{1}{2}Pf$ is

$$K = \frac{S^2Il}{6Ec^2} = \frac{S^2}{2E}\frac{r^2}{3c^2}Al,$$

in which I, the moment of inertia of the section, has been replaced by its equivalent Ar^2, where A is the section area and r is its radius of gyration (Art. 33).

In like manner for a simple beam under a full uniform load the elastic resilience is found to be

$$K = \frac{S^2}{2E} \cdot \frac{5r^2}{3c^2}Al,$$

which is five times as great as for the concentrated load.

These expressions show that the elastic resilience of beams of similar cross-sections are proportional to their volumes. For rectangular sections where the depth is d, the value of c is $\frac{1}{2}d$, and the value of r^2 is $\frac{1}{12}d^2$; thus r^2/c^2 is $\frac{1}{3}$. Hence the resilience of a rectangular beam under a load at the middle is one-ninth, and under a uniform load five-ninths of that of a rectangular bar under tensile stress.

Prob. 53. A heavy 20-inch steel **I** beam of 24 feet span is stressed under a rolling load from 500 up to 8 000 pounds per square inch. Compute the resilience.

(Ans. 26 700 inch-pounds.)

ART. 54. ULTIMATE RESILIENCE.

The ultimate resilience of a body is equal to the external work required to produce rupture. The ultimate resilience greatly surpasses the elastic resilience, it being for wrought iron and steel sometimes five hundred times as large. It is not possible, however, to establish

a formula by which the ultimate resilience can be computed, because the law of increase of the deformations beyond the elastic limit is unknown.

If a diagram be made showing the increase of elongation with stress, as in the figure of Art. 4, the abscissas indicating the elongations and the ordinates the stresses, then the area included between the curve and the axis of elongations represents the ultimate resilience for one cubic inch of the material. The total ultimate resilience is then found by multiplying this by the volume in cubic inches.

In Art. 14 it was remarked that the product of the ultimate strength and ultimate elongation is an index of the quality of wrought iron and steel. This is so because it is a rough measure of the ultimate resilience. A measure which more closely fits the area given by a stress-diagram is

$$k = \tfrac{1}{3}s(S_e + 2S_t),$$

in which S_e is the elastic limit, S_t the ultimate tensile strength, and s the ultimate unit-elongation. For example, take a wrought iron specimen where $S_e = 25\,000$ and $S_t = 55\,000$ pounds per square inch, while $s = 13$ per cent $= 0.13$; then $k = 8\,100$ inch-pounds is the ultimate resilience for one cubic inch of the material.

Prob. 54. Show from the values given in Arts. 2 and 4 that the average ultimate resilience of timber in tension is about 50 per cent greater than that of cast iron.

ART. 55. SUDDEN LOADS.

When a tension is gradually applied to a bar it increases from 0 up to its final value, while the elongation increases from 0 to e and the unit-stress increases

from 0 to S. A 'sudden load' is one which has the same intensity from the beginning to the end of the elongation; this elongation being produced, the bar springs back, carrying the load with it, and a series of oscillations results, until finally the bar comes to rest with the elongation e. The temporary elongation produced is greater than e, and hence also the temporary stresses produced are greater than S.

Let P be a suddenly applied load and y the temporary elongation produced by it; the external work performed during its application is Py. Now let Q be the internal stress corresponding to the elongation y; this increases gradually and uniformly from 0 up to Q, and hence its resilience or internal work is $\frac{1}{2}Qy$. But as internal work must equal external work,

$$\tfrac{1}{2}Qy = Py, \quad \text{or} \quad Q = 2P;$$

that is, the sudden load P produces a temporary internal stress equal to $2P$.

Now after the oscillations have ceased, the bar comes to rest under the steady load P and has the elongation e. If the elastic limit of the material be not exceeded, corresponding elongations are proportional to their stresses; thus

$$\frac{y}{e} = \frac{Q}{P} = 2, \quad \text{or} \quad y = 2e;$$

that is, the sudden load produces a temporary elongation double that caused by the same load when gradually applied.

If A be the section area of the bar the unit-stress S under the gradual load is P/A, and the temporary unit-stress produced under the sudden load is $2P/A$ or $2S$.

The unit-stresses temporarily produced by sudden loads are hence double those caused by steady loads. It is for this reason that factors of safety are taken higher for variable loads than for steady ones.

Prob. 55. A simple beam of wrought iron, 2×2 inches and 18 inches long, is to be loaded with 3 000 pounds at the middle. Show that the beam will be unsafe if this be applied suddenly.

ART. 56. STRESSES DUE TO IMPACT.

Impact is said to be produced in a bar or beam when a load falls upon it from a certain height. The temporary stresses and deformations in such a case are greater than for sudden loads, and may often prove very injurious to the material. If the elastic limit be not exceeded, it is possible to deduce an expression showing the laws that govern the stresses produced by the impact. This will here be done only for the case of impact on the end of a bar.

If the load P falls from the height h upon the end of a bar and produces the momentary elongation y, the work performed is $P(h + y)$. The stress in the bar increases gradually and uniformly from 0 up to the value Q, so that the resilience or internal work is $\frac{1}{2}Qy$. Hence there results

$$\tfrac{1}{2}Qy = P(h + y).$$

Also, if e be the elongation due to the static load P, the law of proportionality of elongation to stress gives

$$\frac{y}{e} = \frac{Q}{P}.$$

By solving these equations the values of Q and y are

$$Q = P\left(1 + \sqrt{\frac{2h}{e} + 1}\right),$$

$$y = e\left(1 + \sqrt{\frac{2h}{e} + 1}\right),$$

which give the temporary stress and elongation produced by the impact.

If $h = 0$ these formulas reduce to $Q = 2P$ and $y = 2e$, as found in the last article for sudden loads. If $h = 4e$ they become $Q = 4P$ and $y = 4e$; if $h = 12e$ they give $Q = 6P$ and $y = 6e$. Since e is a small quantity for any bar it follows that a load P dropping from a moderate height upon the end of a bar may produce great temporary stresses and elongations. If these stresses exceed the elastic limit they cause molecular changes which result in brittleness and render the material unsafe.

The above expressions for Q and y are not exact, as the resistance against motion due to the inertia of the material has not been taken into account. In Chapters IX and XI of Mechanics of Materials the subject is discussed far more completely than has been possible here.

Prob. 56. In an experiment upon a spring a steady weight of 15 ounces on the end produced an elongation of 0.4 inches. What temporary elongation would be produced when the same weight is dropped upon the end of the spring from a height of 7 inches? (Ans. 2.8 inches.)

CHAPTER IX.

MISCELLANEOUS APPLICATIONS.

ART. 57. WATER AND STEAM PIPES.

The pressure of water or steam in a pipe is exerted in every direction and tends to tear the pipe apart longitudinally. This external force is resisted by the internal tensile stresses which act in the walls of the pipe normal to the radii. If p be the pressure per square inch exerted by the water or steam, d the diameter and l the length of the pipe, the total pressure P exerted on any diametral plane is $p \,.\, ld$. If t be the thickness of the pipe and S the tensile unit-stress, the total resisting stress will be $S \,.\, 2lt$, if the thickness is not large compared with the diameter. Hence

$$pld = 2Slt, \quad \text{or} \quad pd = 2St,$$

is the formula for discussing water or steam pipes.

Water-pipes are made of cast iron or wrought iron, the former being more common, while for steam the latter is preferable. A water-pipe liable to the shock of water-ram should have a high factor of safety, and in steam-pipes the factors should also be high. The formula above deduced shows that the thickness of a pipe must increase with its diameter, as also with the internal pressure to which it is to be subjected.

For example, let it be required to find the proper thickness for a wrought-iron steam-pipe of 18 inches diameter to resist a steam-pressure of 250 pounds per

square inch. With a factor of safety of 10 the working unit-stress is 5 500 pounds per square inch. Then from the formula

$$t = \frac{pd}{2S} = \frac{250 \times 18}{2 \times 5\,500} = 0.45 \text{ inches,}$$

so that a thickness of $\frac{1}{2}$ inch would probably be selected.

Prob. 57. Find the factor of safety of a cast-iron water-pipe 12 inches in diameter and $\frac{5}{8}$ inches thick under a pressure of 130 pounds per square inch.

ART. 58. RIVETED LAP JOINTS.

When two plates are joined together by rivets and the plates then subject to tension, there is brought a shear upon the rivets which tends to cut them off. A riveted lap joint is one where one plate simply laps over the other.

For a lap joint with a single row of rivets, let P be the tensile stress which is transmitted from one plate

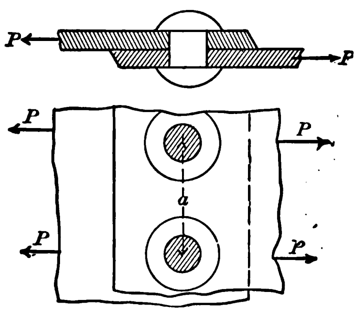

FIG. 37.

to another by means of one rivet; let a be the pitch of the rivets, or the distance from the center of one

rivet to the center of the next; let d be the diameter of a rivet and t the thickness of the plate. The plate tends to tear apart on the section area $(a - d)t$, while the rivet tends to shear off on the section area $\frac{1}{4}\pi d^2$. Accordingly, if S_t and S_s be the unit-stresses for tension and shear,

$$P = t(a - d)S_t, \quad P = \tfrac{1}{4}\pi d^2 S_s,$$

are formulas for the discussion of this case.

For example, a steel water-pipe 30 inches in diameter has a longitudinal rivet seam with one row of rivets, the diameter of the rivets being $\frac{3}{4}$ inches, their pitch 2 inches, and the thickness of the plate $\frac{1}{2}$ inches. If the interior water-pressure be 130 pounds per square inch, what are the unit-stresses in tension and shear? Here the total pressure on a diametral plane of length equal to the pitch is $P = 130 \times 2 \times 30 = 7\,800$ pounds. Then for tension on the plate

$$S_t = \frac{7\,800}{\frac{1}{2}(2 - \frac{3}{4})} = 12\,500 \text{ pounds per square inch,}$$

and for shear on the rivet

$$S_s = \frac{7\,800}{0.785 \times \frac{9}{16}} = 17\,700 \text{ pounds per square inch.}$$

The unit-stress for tension is perhaps not too high, but that for shear is about double of the proper value; this is because the joint is not properly designed.

When two rows of rivets are used, these are staggered, so that the rivets in one row come opposite the middle of the pitches in the other row. Here the tension P is distributed over two rivet sections instead of one, and

$$P = t(a - d)S_t, \quad P = \tfrac{1}{2}\pi d^2 S_s,$$

are the formulas for investigation. Thus for the data of the above example, if there are two rows of rivets,

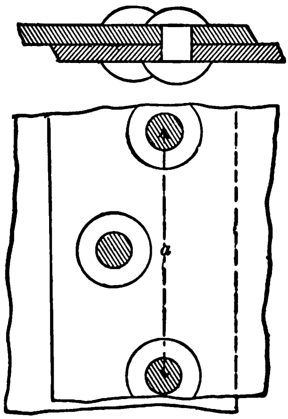

FIG. 38.

S_t is 12 500 pounds per square inch, and $S_s = 8\,900$ pounds per square inch, which are good working values for mild steel if the pipe is not subjected to the shock of water-ram.

The working unit-stress for shear should be about three-fourths of that for tension, or $S_s = \frac{3}{4}S_t$. Equating the above values of P under this condition gives a joint whose security in tension is the same as that in shear; thus

$$a = d + 0.59\frac{d^2}{t} \qquad\qquad a = d + 1.18\frac{d^2}{t},$$

the first for single lap riveting and the second for double lap riveting. These are approximate rules for finding the pitch when the thickness of plates and diameter of rivets are given.

Prob. 58. A steel water-pipe 30 inches in diameter has

rivets $\frac{3}{4}$ inches in diameter and plates $\frac{1}{2}$ inches thick. If double riveting is used, what should be the pitch of the rivets?

ART. 59. RIVETED BUTT JOINTS.

When two plates butt together cover plates are used on one or both sides; if on both sides the covers are one-half the thickness of the main-plate. The shear on each rivet is here divided between the upper and the

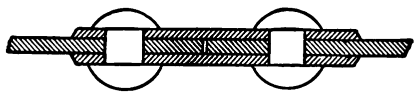

FIG. 39.

lower cross-section, this being called a case of double shear. Thus if P be the tension which is transmitted through one rivet, d the diameter of a rivet, a the pitch, and t the thickness of the main plate,

$$P = t(a - d)S_t, \qquad P = 2 \cdot \tfrac{1}{4}\pi d^2 S_s,$$

which are the same as for two rows of lap riveting.

The 'efficiency' of a riveted joint is the ratio of the strength of the joint to that of the solid plate. If the joint be designed so as to be of equal strength in tension and shear this efficiency is

$$\frac{t(a - d)S_t}{taS_t} = 1 - \frac{d}{a}.$$

Thus if the rivets be $\frac{3}{4}$ inches in diameter and the pitch be 2 inches the efficiency is $1 - \frac{3}{8} = 0.625$, that is, the riveted joint has only 62.5 per cent of the strength of the solid plate. Single lap riveting has usually an efficiency of about 60 per cent, while double lap riveting and common butt riveting has from 70 to

75 per cent. By using three or more rows of rivets efficiencies of over 80 per cent can be secured.

When a joint is not of equal strength in tension and shear there are two efficiencies, one being the ratio of the tensile strength of the joint to that of the solid plate, and the other the ratio of the shearing strength to that of the solid plate. The least of these is the true efficiency.

Prob. 59. A butt joint with two cover-plates has the main plate $\frac{1}{2}$ inches thick, the rivets $\frac{7}{8}$ inches in diameter, and the pitch of the rivets $2\frac{7}{8}$ inches. Compute the efficiency.

ART. 60. STRESSES DUE TO TEMPERATURE.

A bar which is free to move elongates when the temperature rises and shortens when it falls. But if the bar be under stress so that it cannot elongate or contract, the change in temperature produces a certain unit-stress. This unit-stress is that which would cause a change of length equal to that produced in the free bar by the change in temperature.

The coefficient of linear expansion is the elongation of a bar of length unity under a rise of temperature of one degree. For the Fahrenheit degree the average values of the coefficients of expansion are as follows:

For brick and stone, $C = 0.000\,00\,50$;
For cast iron, $C = 0.000\,00\,62$;
For wrought iron, $C = 0.000\,00\,67$;
For steel, $C = 0.000\,00\,65$.

Thus a free bar of cast iron 1000 inches long will elongate 0.0062 inches for a rise of one degree, and 0.62 inches for a rise of 100 degrees.

The elongation of a bar of length unity for a change of t degrees is hence $s = Ct$. But (Art. 44) the unit-stress due to the unit-elongation s is $S = sE$, where E is the coefficient of elasticity. Therefore

$$S = CtE$$

is the unit-stress produced by a change of t degrees on a bar which is fixed. If the temperature rises S is compression, if the temperature falls S is tension.

For example, consider a wrought-iron rod which is used to tie together two walls of a building and which is screwed up to a stress of 10 000 pounds per square inch. If the temperature falls 50 degrees there is produced a tensile unit-stress,

$$S = 0.0000067 \times 50 \times 25\,000\,000 = 8\,400,$$

and hence the total stress in the rod is 18 400 pounds per square inch. If the temperature rises 50 degrees the stress in the bar is reduced to 1 600 pounds per square inch. In all cases the unit-stresses due to temperature are independent of the length and section area of the bar.

Prob. 60. A cast-iron bar 6 feet long and 4 × 4 inches in section is confined between two immovable walls. What pressure is brought on the walls by a rise of 40 degrees in temperature? (Ans. 59 500 pounds.)

ART. 61. SHRINKAGE OF HOOPS.

A hoop or tire is frequently turned with the inner diameter slightly less than that of the cylinder or wheel upon which it is to be placed. The hoop is then expanded by heat and placed upon the cylinder, and upon cooling it is held firmly in position by the radial

stress produced. This radial stress causes tension in the hoop.

Let D be the diameter of the cylinder upon which the hoop is to be shrunk and d the interior diameter to which the hoop is turned. If the thickness of the hoop is small, D will be unchanged by the shrinkage and d will be increased to D. The unit-elongation of the hoop is then $s = (D - d)/d$, and hence the unit-stress produced is

$$S = sE = \frac{D-d}{d}E,$$

where E is the coefficient of elasticity of the material.

A common rule in steel hoop shrinkage is to make $D - d$ equal to $\frac{1}{1500}d$; that is, the cylinder is turned so that its diameter is $\frac{1}{1500}$th greater than the inner diameter of the hoop. Accordingly the tangential unit-stress which occurs in the hoop after shrinkage is $30\,000\,000/1\,500 = 20\,000$ pounds per square inch.

When the hoop is thick the above rule is not correct, for a part of the stress produced by the shrinkage causes the diameter of the cylinder to be decreased. The rules for this case are complex ones, and cannot be developed in an elementary text-book; they will be found in Chapter XIV of Mechanics of Materials.

Prob. 61. Upon a cylinder 18 inches in diameter a thin wrought-iron hoop is to be placed. The hoop is turned to an inner diameter of 17.99 inches and then shrunk on. Compute the tensile unit-stress in the hoop.

ART. 62. SHAFT COUPLINGS.

Let a shaft be in two parts which are connected by a flange coupling. In the figure A shows the end view and B the side view of the coupling. The flanges

of the coupling are connected by bolts which are brought into shearing stress in transmitting the torsion from one part of the shaft to the other.

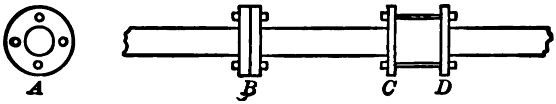

FIG. 40.

Let the shaft be solid and of diameter D, let there be n bolts of diameter d, and let h be the distance from the center of the shaft to the center of a bolt. If D and d be assumed, as also the distance h, then, as shown in Art. 124 of Mechanics of Materials, the formula

$$n = \frac{(d + 2h)D^3}{(d^2 \times 8h^2)d^2}$$

gives the number of bolts required in order that the strength of the bolts may be the same as the strength of the shaft.

For example, let $D = 8$ inches, $d = 1$ inch, and $h = 12$ inches, then the formula gives $n = 11.1$, so that 12 bolts should be used. If $D = 8$ inches, $d = 1\frac{1}{2}$ inches, and $h = 12$ inches, the formula gives $n = 5.01$, so that five or six bolts should be used.

The case shown at CD in the above figure is one that should never occur in practice, because here the bolts are subject to a bending stress as well as to the shearing stresses due to the torsion. It is clear that this bending stress will increase with the length between the flanges, and that the bolts should be greater in diameter than for the case of pure shearing.

Prob. 62. A solid steel shaft 16 inches in diameter

transmits 16 000 horse-powers at 50 revolutions per minute. Design a flange coupling for this shaft.

ART. 63. RUPTURE OF BEAMS AND SHAFTS.

The formulas (4) and (6) deduced in Arts. 25 and 40 for the discussion of beams and shafts are only valid when the unit-stress S is less than the elastic limit of the material. Formula (4) can, however, be used for cases of rupture of beams, provided that S be taken as a certain quantity intermediate between the ultimate and tensile strengths of the material. This quantity, which is called the 'modulus of rupture,' has been determined by breaking beams and then computing S from the formula. In like manner formula (6) can be used for the rupture of shafts if the proper modulus of rupture, as found by experiment, be used instead of the ultimate shearing strength.

The following table gives average values of the modulus of rupture as determined by testing beams and

TABLE XI. MODULUSES OF RUPTURE.

Material.	For Beams.		For Shafts.	
	Pounds per Square Inch.	Kilos per Square Centimeter.	Pounds per Square Inch.	Kilos per Square Centimeter.
Timber	9 000	630	2 000	140
Stone	2 000	140		
Cast Iron	35 000	2 450	25 000	1 750
Wrought Iron			50 000	3 500
Steel	125 000	8 750	75 000	5 250

columns to destruction. Wrought iron and mild steel have no proper modulus of rupture when used as

beams, since they continually bend and do not break however great the load may be. By the use of these values formulas (4) and (6) may be applied to the solution of numerous problems relating to the rupture of beams and shafts.

For example, let it be required to find the size of a square cast-iron simple beam of 6 feet span that will rupture under its own weight. Let x be the side of the square. The weight of a cast-iron bar x square inches in section area and one yard long is $9.4x$ pounds; thus the weight of the beam is $18.8x$ pounds. The bending moment is $\frac{1}{8}Wl$ or $169.2x$ pound-inches. The value of c is $\frac{1}{2}x$ and that of I is $\frac{1}{12}x^4$. Then, by formula (4),

$$\frac{35\,000 \times x^4}{24x} = 169.2x,$$

and the solution of this gives $x = 0.117$ inches.

It is to be noted, when formulas (4) and (6) are used for cases of rupture, that they are entirely empirical and have no rational basis.

Prob. 63. What force P acting at the end of a lever 4 feet long will twist asunder a steel shaft 1.4 inches in diameter? (Ans. 840 pounds.)

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